

Multiple Trait Selection

Multiple Trait Breeding Objectives

Multiple Trait breeding values

Selection on multiple traits

Predicting multi-trait selection response

Manipulating multi-trait response

Multiple Trait Breeding Objectives

- Animals have many characteristics



» Do we want to improve them all ?

Issues with Multiple Trait selection

- We have to spread our selection efforts over several traits
- Not all traits are equally important economically
- Not all traits are equally heritable
- There are correlations between traits
 - Selection for one trait gives also a correlated response for other traits
- How to weight optimally the different traits?

Multiple Trait Selection

- Setting the Breeding Objective
- Defining MT Selection Weights
- Predicting MT Selection Response
- Manipulating MT Selection Response

Introduction to Breeding Objectives

Where do we want to go?

Many possible traits to improve

Many possible traits to record

What is the value of improving different traits?

How do we combine information on different traits to get to where we want to go?

What is a Breeding Objective?

Overall statement about what we want to achieve

e.g.

- Maximise profit
- Minimise costs
- Maximise bad temper and ugliness
- Maximise gross national happiness (Kingdom of Bhutan)



What is an Aggregate Genotype?

A function of genetically controlled traits that contribute value to the breeding objective, that, if maximised (or minimised) will achieve the breeding objective.

$$H = v_1g_1 + v_2g_2 + v_3g_3 \dots \text{etc}$$

Where:

v_i are ***economic weights***

g_i are additive genetic values of an individual animal

Note:
linearity assumed here
(big assumption?)

What is a Selection Index?

A function of genetically controlled phenotypes (or EBV) that if maximised will maximise the aggregate genotype which will achieve the breeding objective.

$$I = b_1x_1 + b_2x_2 + b_3x_3 \dots \text{etc}$$

Where b_i are ***selection index weights*** and x_i are the phenotypes (or EBV) of an individual animal

The Logical Process

Define breeding objective



Develop the aggregate genotype



Develop selection index

Defining an Aggregate Genotype

$$H = v_1g_1 + v_2g_2 + v_3g_3\dots\dots\text{etc}$$

Which traits are included?

- All ***genetically controlled*** traits that contribute to profit
- Exclude traits that only indirectly associated with profit (eg **conformation traits**)
- Can exclude traits with very little genetic variation (**not the same as low heritability**)

Note: Not all traits in H may be measured, and there is limited cost associated with including them in H.

Defining an Aggregate Genotype

$$H = v_1g_1 + v_2g_2 + v_3g_3 \dots \text{etc}$$

An **economic weight**, v_i , is a partial weight; it is the value of increasing trait i by one unit when all other traits remain unchanged.

An **economic weight** is the rate of change in profit as the genetic mean of the trait changes, when all other traits remain unchanged

Selection Index / Selection Criteria Traits

$$I = b_1x_1 + b_2x_2 + b_3x_3 \dots \text{etc}$$

Which traits are included?

- Traits that are relatively easy to measure
- Traits that are included in H
- Traits that are correlated with traits in H

Note: These traits require measurement (=cost) so need to do cost-benefit on whether worth while to include them

Selection Index Concept

Single trait objective

Selection Criteria



Breeding Objective

Own performance X_{11}

Performance on relatives X_{12}

Breeding Value (g_1)

X_{12} = phenotypic measurement
(2nd source on Trait 1)

g_1 = breeding value for trait 1

Selection Index (multiple regression)

$$EBV_1 = b_1 X_{11} + b_2 X_{12} + \dots$$

Selection Index Concept

Single trait objective

Selection Criteria



Breeding Objective

Own performance X_{11}

Performance on relatives X_{12}

Correlated Trait X_{21}

Breeding Value (g_1)

$g_1 = \text{breeding value for trait 1}$

Selection Index (multiple regression)

$$EBV_1 = b_1 X_{11} + b_2 X_{12} + b_3 X_{21} + \dots$$

Selection Index Concept

Multi trait objective

Selection Criteria



Breeding Objective

Own performance X_{11}

Performance on relatives X_{12}

Correlated Trait X_{21}

$$H = v_1g_1 + v_2g_2$$

Selection Index (multiple regression)

$$\text{MTIndex} = b_1X_{11} + b_2X_{12} + b_3X_{21} + \dots$$

Selection Index Concept

Single trait objective

Selection Criteria



Breeding Objective

Own performance X_{11}

Performance on relatives X_{12}

Correlated Trait X_{21}

Breeding Value (g_1)

$g_1 = \text{breeding value for trait 1}$

Selection Index (multiple regression)

$$EBV_1 = b_1 X_{11} + b_2 X_{12} + b_3 X_{21} + \dots$$

Selection Index Concept

Single trait objective

Selection Criteria



Breeding Objective

Own performance X_{11}

Performance on relatives X_{12}

Correlated Trait X_{21}

Breeding Value (g_1)

Breeding Value (g_2)

$g_2 = \text{breeding value for trait 2}$

Selection Index (multiple regression)

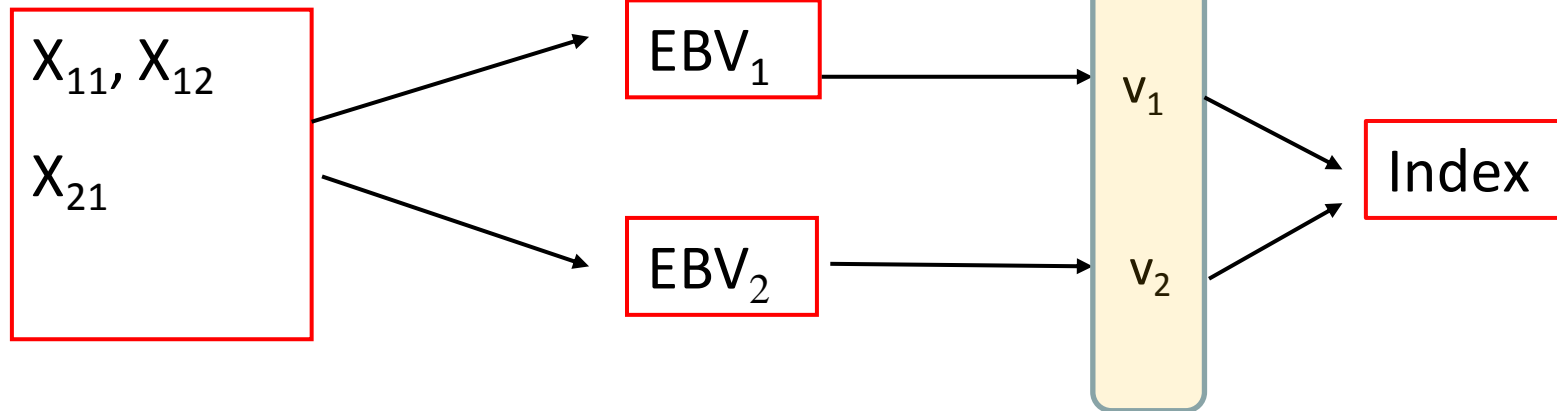
$$EBV_2 = b_1 X_{11} + b_2 X_{12} + b_3 X_{21} + \dots$$

Selection Index Concept

Multi trait objective

Phenotypic data \Rightarrow Breeding Obj Traits \Rightarrow Breeding Obj Overall Merit

weights

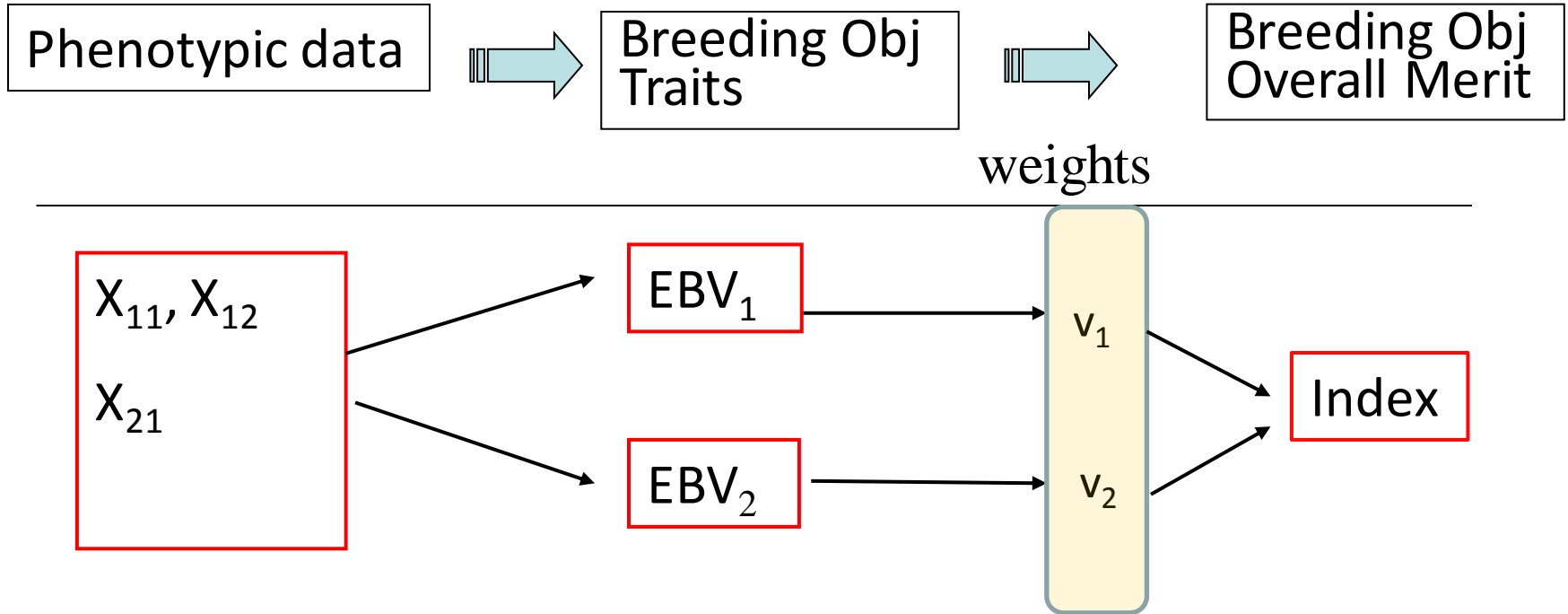


Selection Index (multiple regression)

$$\text{Index} = v_1 EBV_1 + v_2 EBV_2$$

Selection Index Concept

Multi trait objective



Selection Index (multiple regression)

$$\text{Index} = v_1 EBV_1 + v_2 EBV_2 = v_1 [b_{11}X_{11} + b_{12}X_{12} + b_{13}X_{21} + \dots] + v_2 [b_{21}X_{11} + b_{22}X_{12} + b_{23}X_{21} + \dots]$$

$$\text{Index} = b_1X_{11} + b_2X_{12} + b_3X_{21} + \dots$$

Some formal definitions

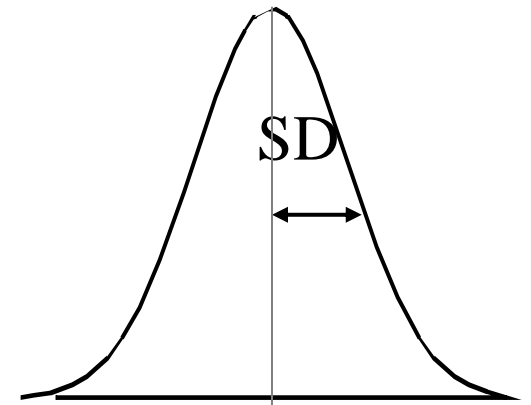
Aggregate Genotype / Breeding Goal

$$H = v_1g_1 + v_2g_2 + v_3g_3 \dots \text{etc} = v'g$$

$$\text{Var}(H) = \sigma_H^2 = v'Cv \quad \text{where } C = \text{var}(g)$$

σ_H = SD of breeding objective = SD in profit

$\sigma_H \cong$ \$10 (sheep)
 \$30 (beef)
 \$50 (dairy)



SD of genetic merit
for 'profit'

Some formal definitions

$$H = v_1g_1 + v_2g_2 + v_3g_3 \dots \text{etc} = v'g$$

$$\text{Var}(H) = \sigma_H^2 = v' Cv$$

$$I = b_1x_1 + b_2x_2 + b_3x_3 \dots \text{etc}$$

$$\text{Var}(I) = b' P b$$

where $P = \text{var}(x)$

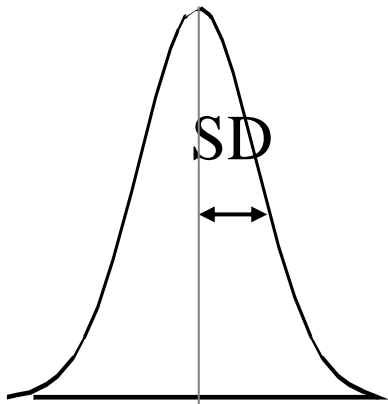
$\sigma_I = \text{SD of Index}$

$$\text{cov}(x, g) = G$$

G = matrix

$$b = P^{-1} G v$$

V = econ val



SD of \$index

$$\text{accuracy} = \sigma_I / \sigma_H$$

note: $\sigma_I < \sigma_H$

Some basic Quantitative Genetic Theory

Single trait

$$P = A + E$$

→ General Model

$$\text{Var}(P) = \text{var}(A) + \text{var}(E) = V_A + V_E$$

no cov. between A and E

$$\text{cov}(A_i, P_i) = \text{cov}(A_i, A_i) + \text{cov}(A_i, E_i) = V_A$$

if A same animal as P

$$\text{cov}(A_i, P_j) = \text{cov}(A_i, A_j) + \text{cov}(A_i, E_j) = a_{ij} V_A$$

a_{ij} = additive genetic relationship between i and j

$$\text{cov}(P_i, P_j) = a_{ij} V_A$$

as E's are uncorrelated if not same animal

Some basic Quantitative Genetic Theory

Multi trait

$$\text{cov}(P_{1i}, P_{2i}) = r_p \sigma_{P1} \sigma_{P2}$$

phenotypic covariance as

E's are correlated if same animal

$$\text{cov}(P_{1i}, P_{2j}) = a_{ij} r_g \sigma_{g1} \sigma_{g2}$$

genetic covariance as

E's are uncorrelated if not same animal

$$\text{cov}(A_{1i}, P_{2i}) = r_g \sigma_{g1} \sigma_{g2}$$

if A₁ same animal as P₂

$$\text{cov}(A_{1i}, P_{2j}) = a_{ij} r_g \sigma_{g1} \sigma_{g2}$$

if A₁ not same animal as P₂

In general,

when between traits, replace variance by covariance

Types of correlations

- Phenotypic correlations
 - measure association between observed performance
 - Cows that produce more milk tend to have lower fertility
- Genetic correlations
 - measure association between breeding values
 - Bulls with daughters that produce more milk tend to have daughters with lower fertility
 - Due to pleiotropy or linkage (may be +ve or –ve)

Types of correlations

- Phenotypic correlations (r_p)
 - measure association between observed performance
- Genetic correlations (r_g)
 - measure association between breeding values
- Environmental correlations (r_e) $P = A + E$
 - measure association between random environmental effects

- Recall Variances add up $V_P = V_A + V_E$
- Similarly Covariances add up $Cov_P = Cov_A + Cov_E$

But correlations do not add up!

$$r_p \neq r_A + r_E$$

Selection index with more information sources (multiple regression)

p = vector with phenotypes (criteria)

g = breeding objective (single trait BV here)

$$\text{var}(p) = P = \text{matrix} = \begin{bmatrix} \text{var}(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{var}(x_2) \end{bmatrix}$$

$$\text{cov}(p, g) = G = \text{vector} = \begin{bmatrix} \text{cov}(x_1, g) \\ \text{cov}(x_2, g) \end{bmatrix}$$

$$\text{weights: } \mathbf{b} = \mathbf{P}^{-1}\mathbf{G}$$

Selection index with more information sources and with more objective traits (multiple regression)

p = vector with phenotypes (criteria)

H = breeding objective (multiple traits here)

$$= \mathbf{v}_1 \mathbf{g}_1 + \mathbf{v}_2 \mathbf{g}_2$$

$$\text{var}(p) = P = \text{matrix} = \begin{bmatrix} \text{var}(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{var}(x_2) \end{bmatrix}$$

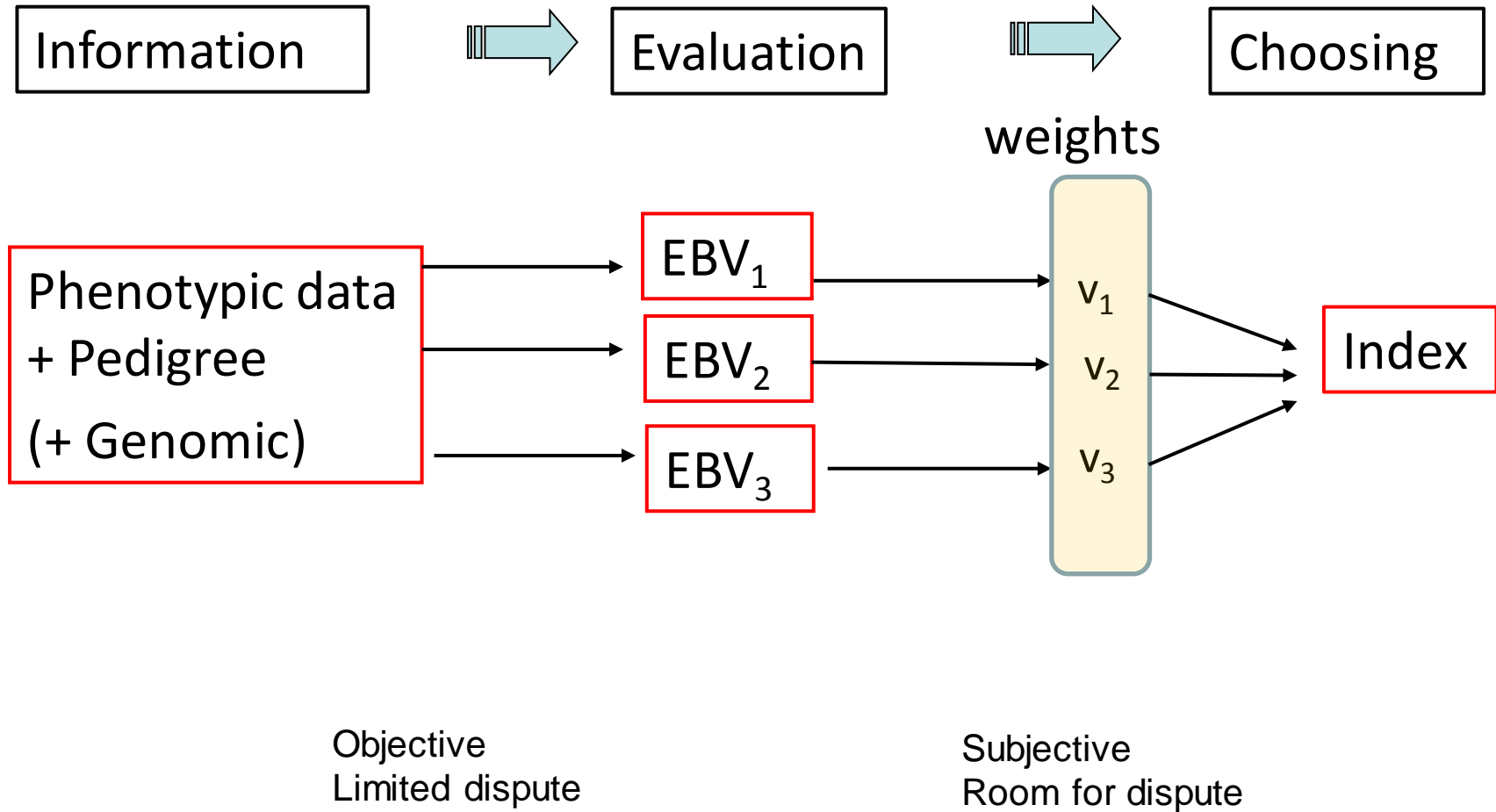
$\text{cov}(p, A) = G = \text{matrix} =$

$$\begin{bmatrix} \text{cov}(x_1, g_1) & \text{cov}(x_1, g_2) \\ \text{cov}(x_2, g_1) & \text{cov}(x_2, g_2) \end{bmatrix}$$

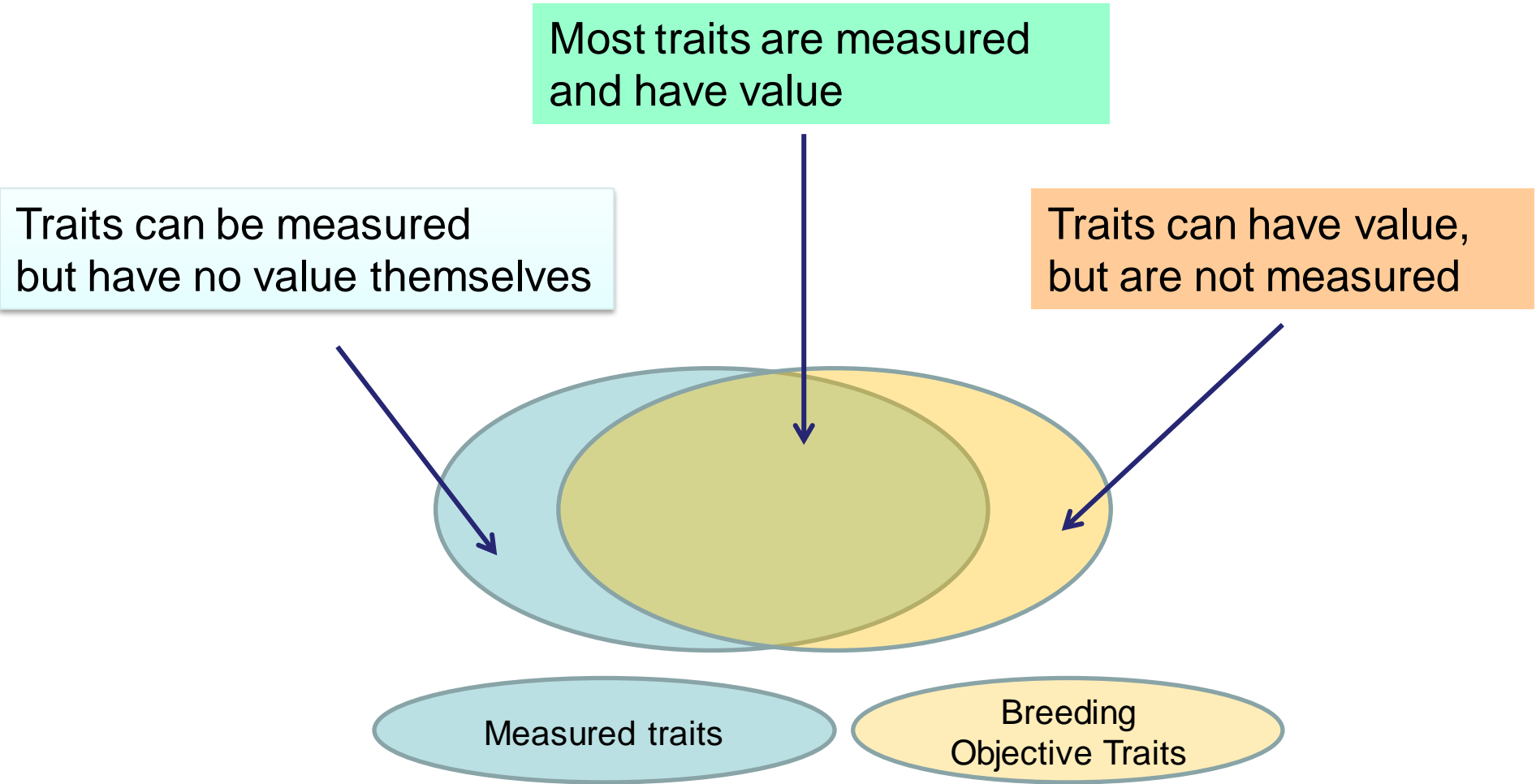
weights: $\mathbf{b} = \mathbf{P}^{-1} \mathbf{G} \mathbf{v}$

\mathbf{v} are economic values

Back to Selection Index Concept



Breeding objective traits and selection criteria traits



Predicting genetic change to multiple trait selection

- Single trait selection response
- Correlated response to selection
- Response to multi trait index selection
 - Predicting response per trait
 - How can multiple trait response be manipulated by varying index weights
 - Can we go anywhere we want?

Example

body weight $h^2 = 0.40$ $\sigma_p = 17 \text{ kg}$

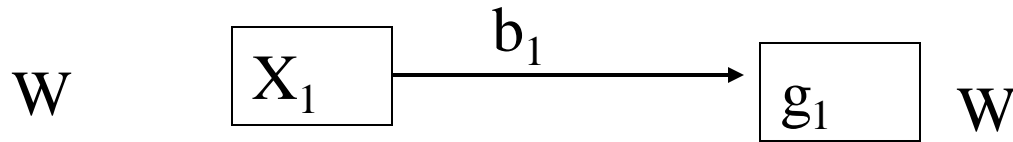
feed intake $h^2 = 0.25$ $\sigma_p = 2 \text{ kg}$

$$r_g = .50$$

$$r_p = 0.20$$

Criteria for selection

Breeding Objective



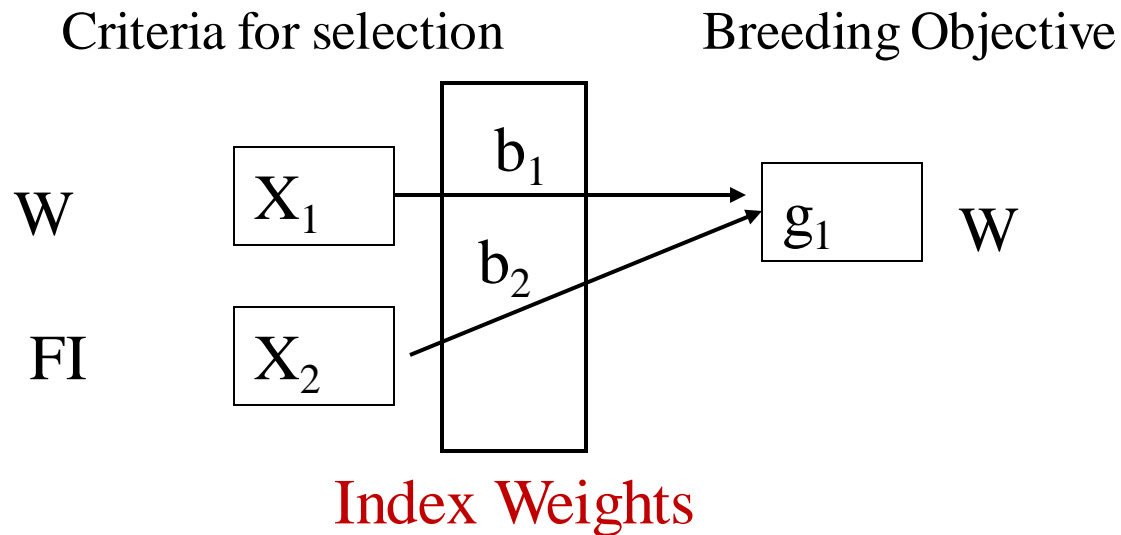
Select on body weight,

Objective is body weight

$$\text{Index} = \text{EBV} = 0.4X_W$$

$$\text{Response} = \mathbf{6.80} \text{ kg Weight}$$

$$\text{Correl. Resp.} = \mathbf{0.32} \text{ kg Feed Intake}$$



Select on body weight + feed intake Objective is body weight

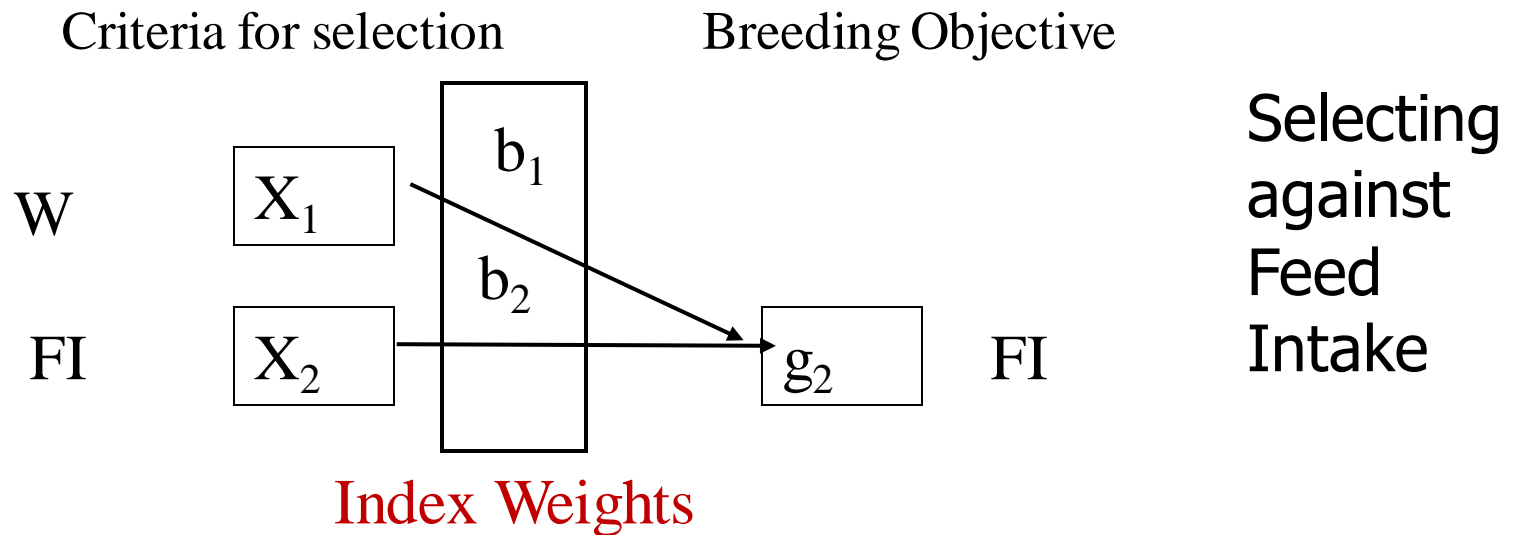
$$\text{Index} = \$EBV = 0.38X_W + 0.69X_{FI}$$

$$R_W = \mathbf{6.93} \text{ kg}$$

$$R_{FI} = \mathbf{0.40} \text{ kg}$$

↙

We select for bigger eaters,
why?



Select on body weight + feed intake

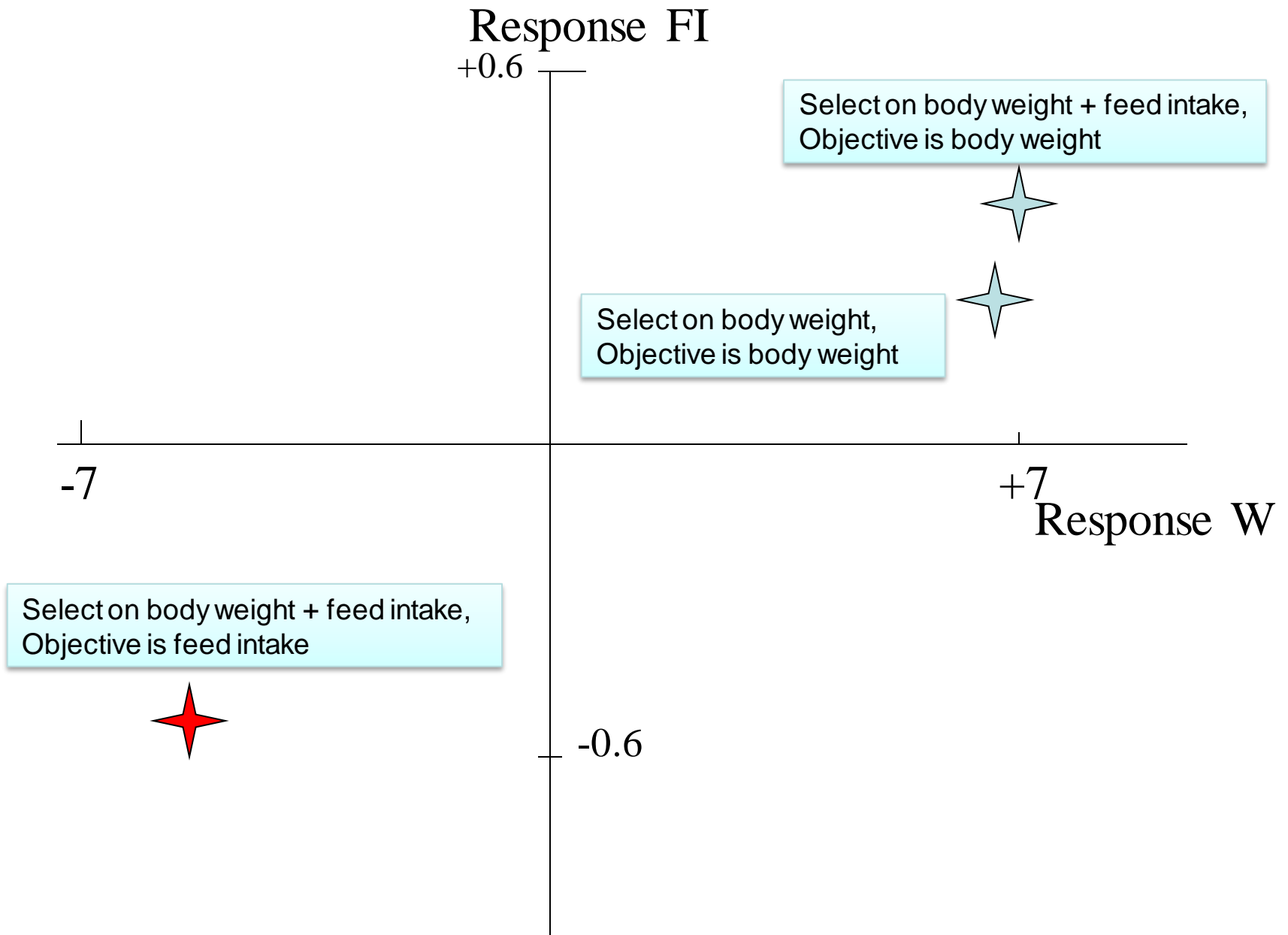
Objective is feed intake

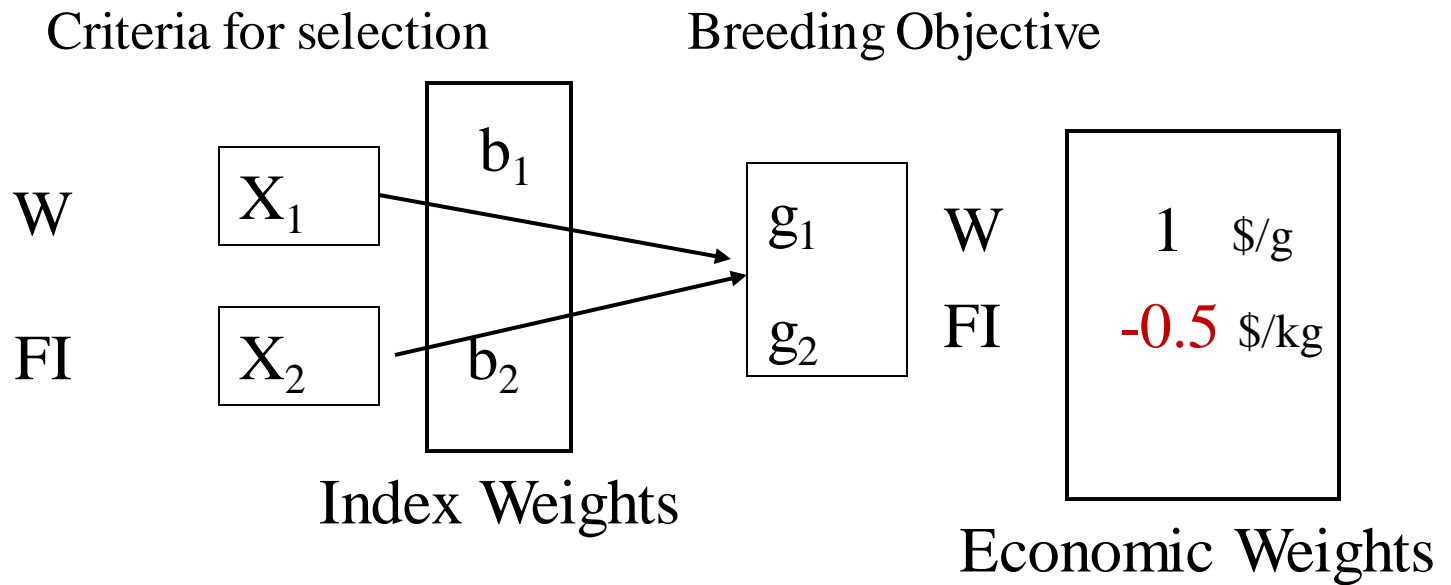
$$\text{Index} = \$EBV = -0.013X_W - 0.23X_{FI}$$

$$R_W = -5.04 \text{ kg}$$

$$R_{FI} = -0.55 \text{ kg}$$

We select against bigger animals, why?





Select on body weight + feed intake

Objective is 1.0.BW -0.5.FI

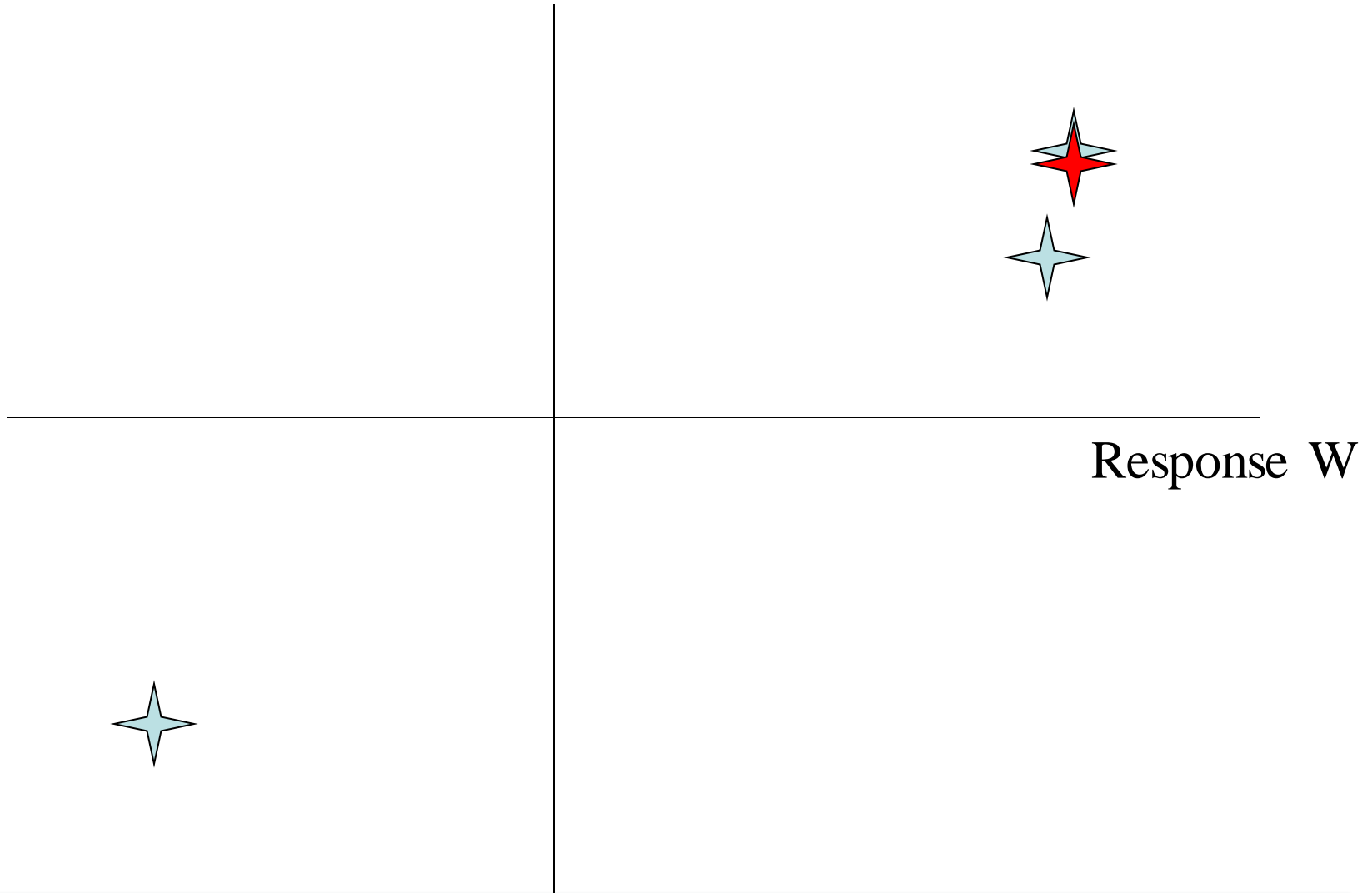
$$\text{Index} = \$EBV = 0.38X_W + 0.67X_{FI}$$

$$R_W = \mathbf{6.93} \text{ kg}$$

$$R_{FI} = \mathbf{0.39} \text{ kg}$$

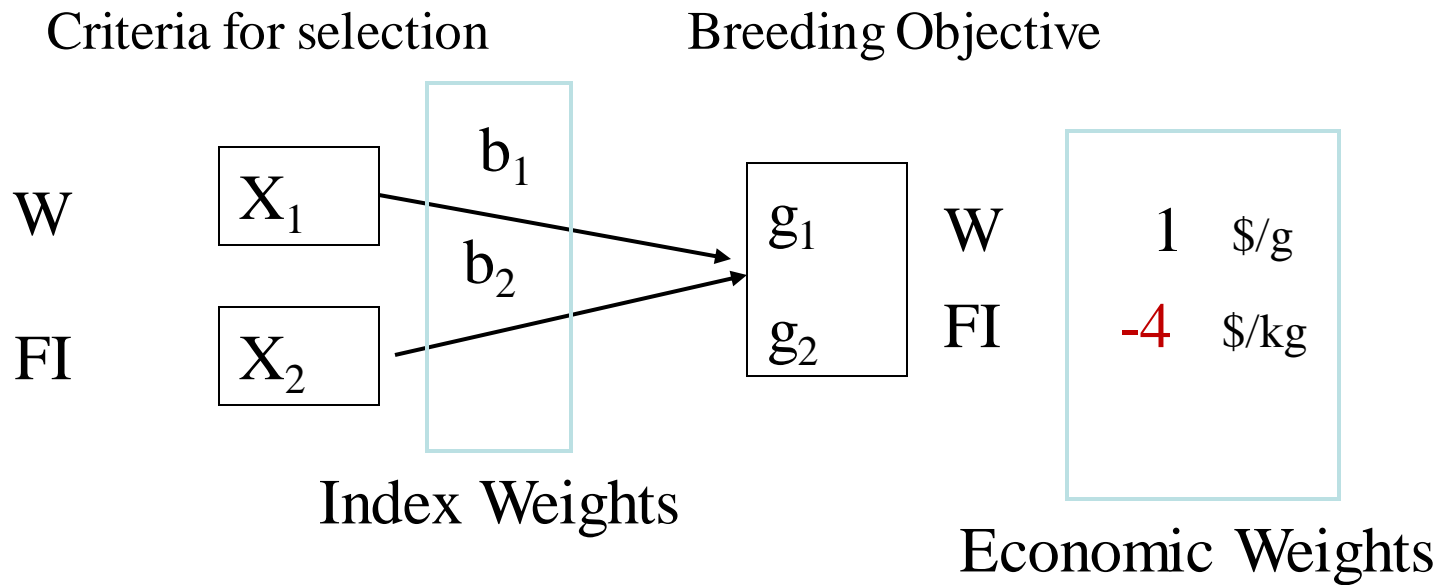
We still select for bigger eaters, why?

Response FI



Select on body weight + feed intake

Objective is $1.0.BW - 0.5.FI$



Select on body weight + feed intake

Objective is $1.0 \cdot BW - 4 \text{ FI}$

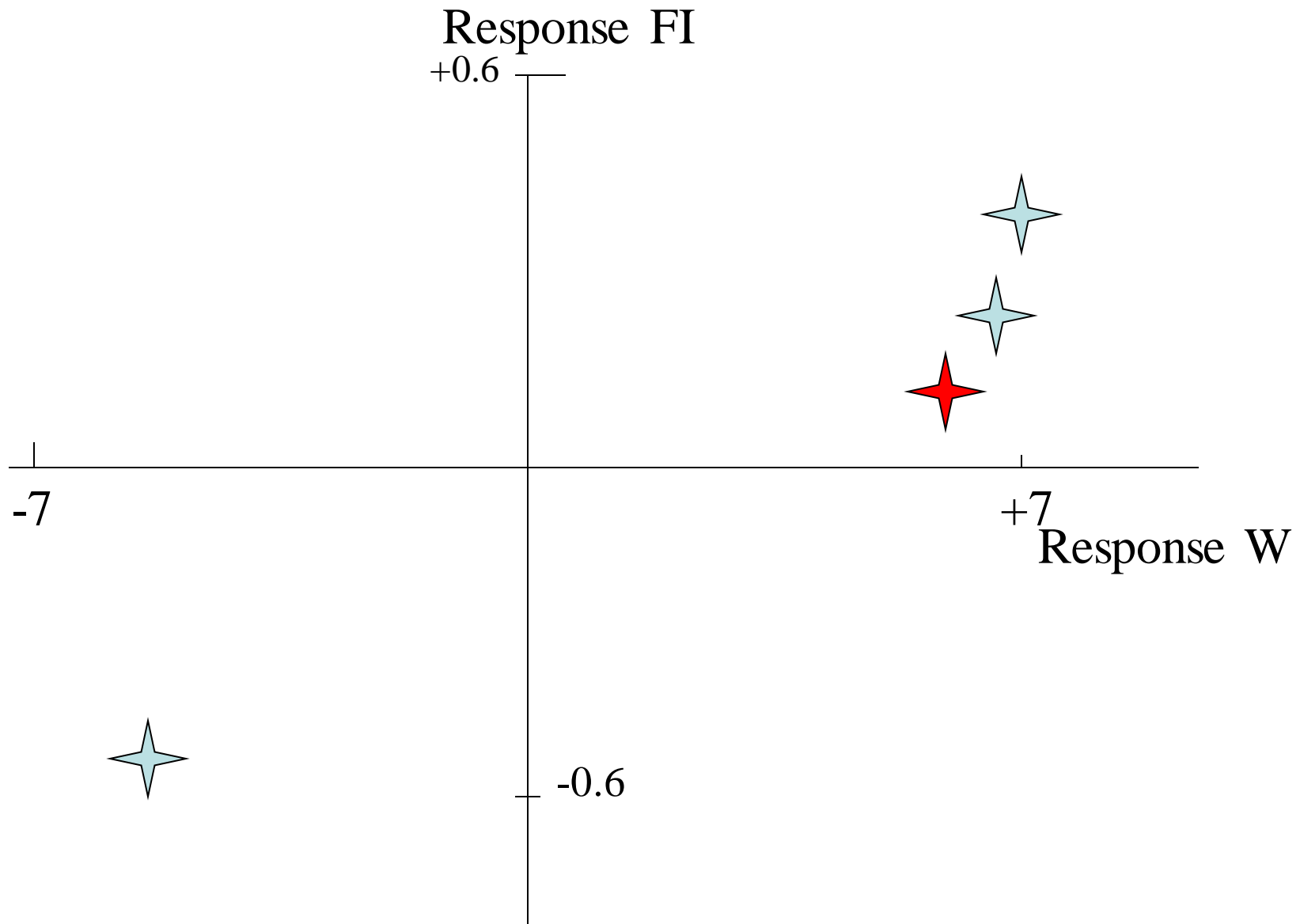
$$\text{Index} = \$EBV = 0.33X_W - 0.22X_{FI}$$

Now we select against bigger eaters, why?

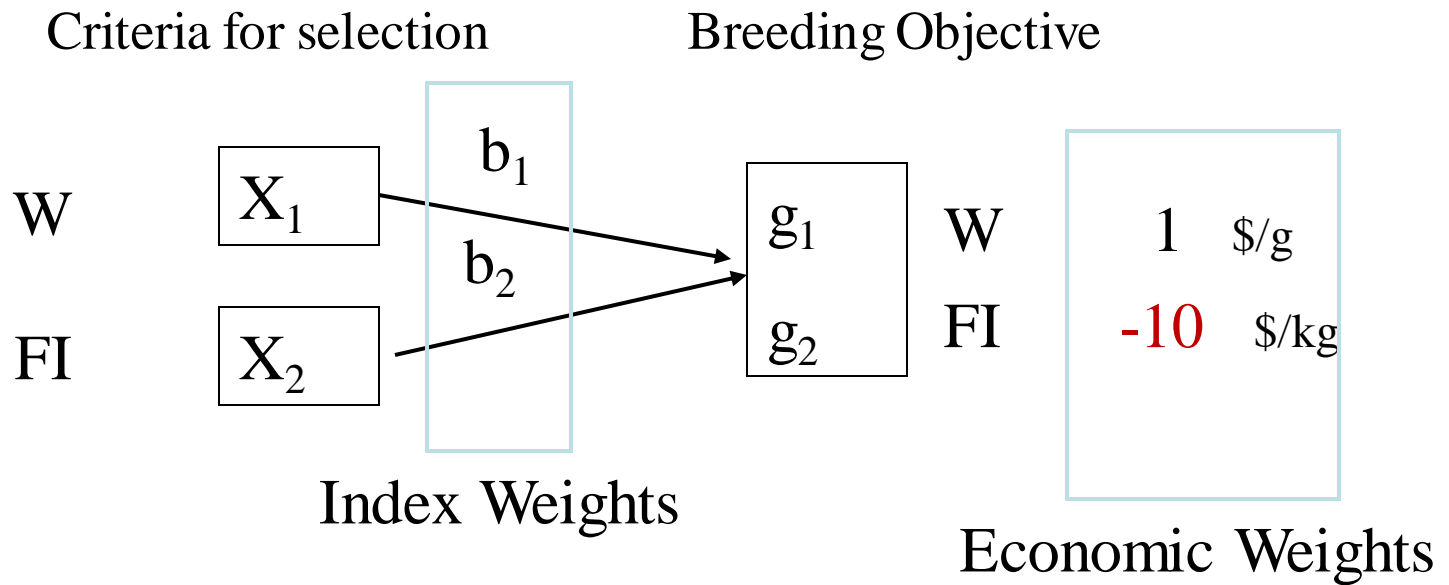
$$R_W = \mathbf{6.68 \text{ kg}}$$

$$R_{FI} = \mathbf{0.28 \text{ kg}}$$

But the response is that we get bigger eaters, why?



Select on body weight + feed intake Objective is $1.0 \cdot BW - 4 \cdot FI$



Select on body weight + feed intake

Objective is $1.0 \cdot BW - 10 \cdot FI$

$$\text{Index} = \$EBV = 0.25X_W - 1.58X_{FI}$$

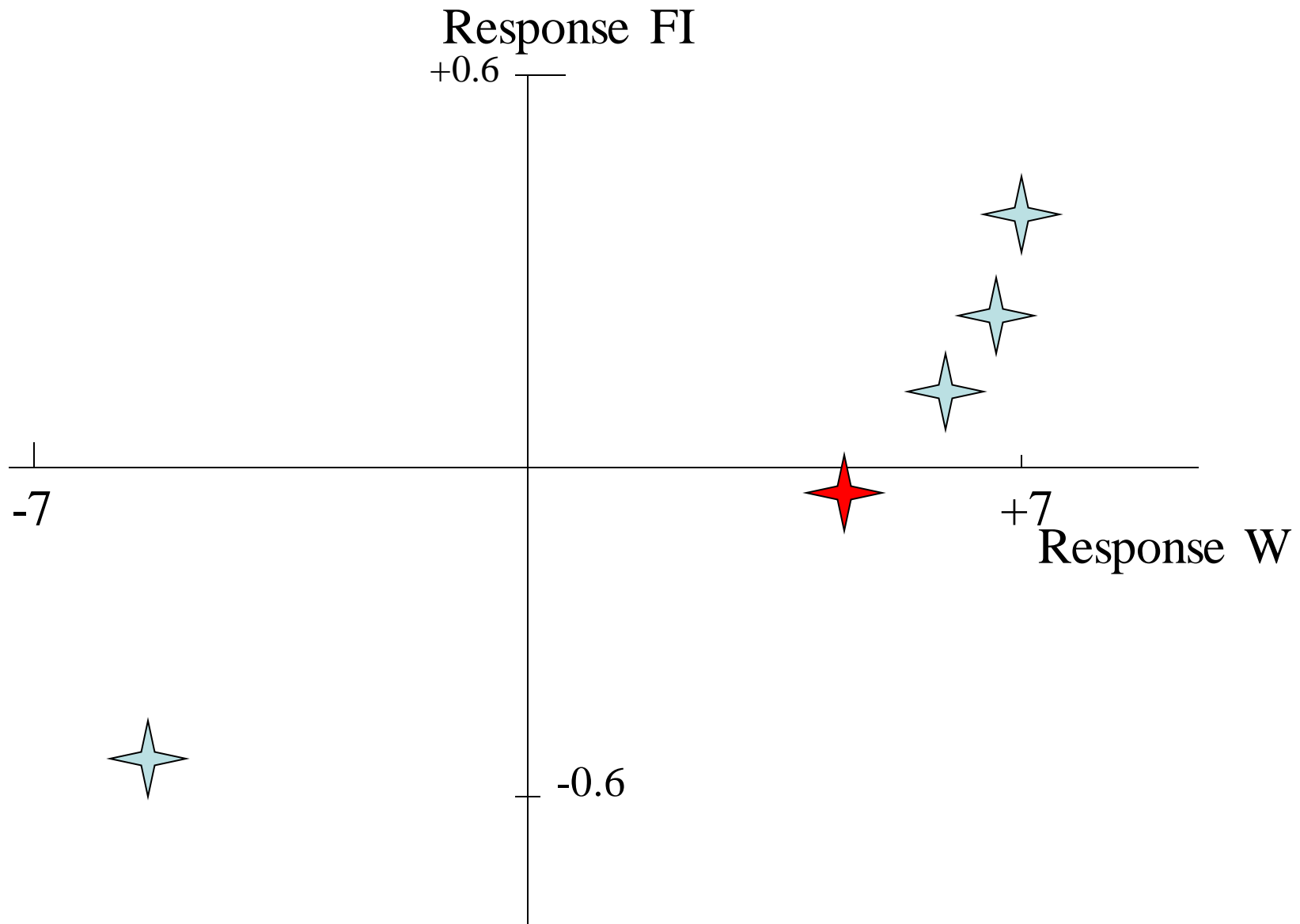
Now we select more strongly against bigger eaters, why?

$$R_W = 4.29 \text{ kg}$$

$$R_{FI} = -0.05 \text{ kg}$$

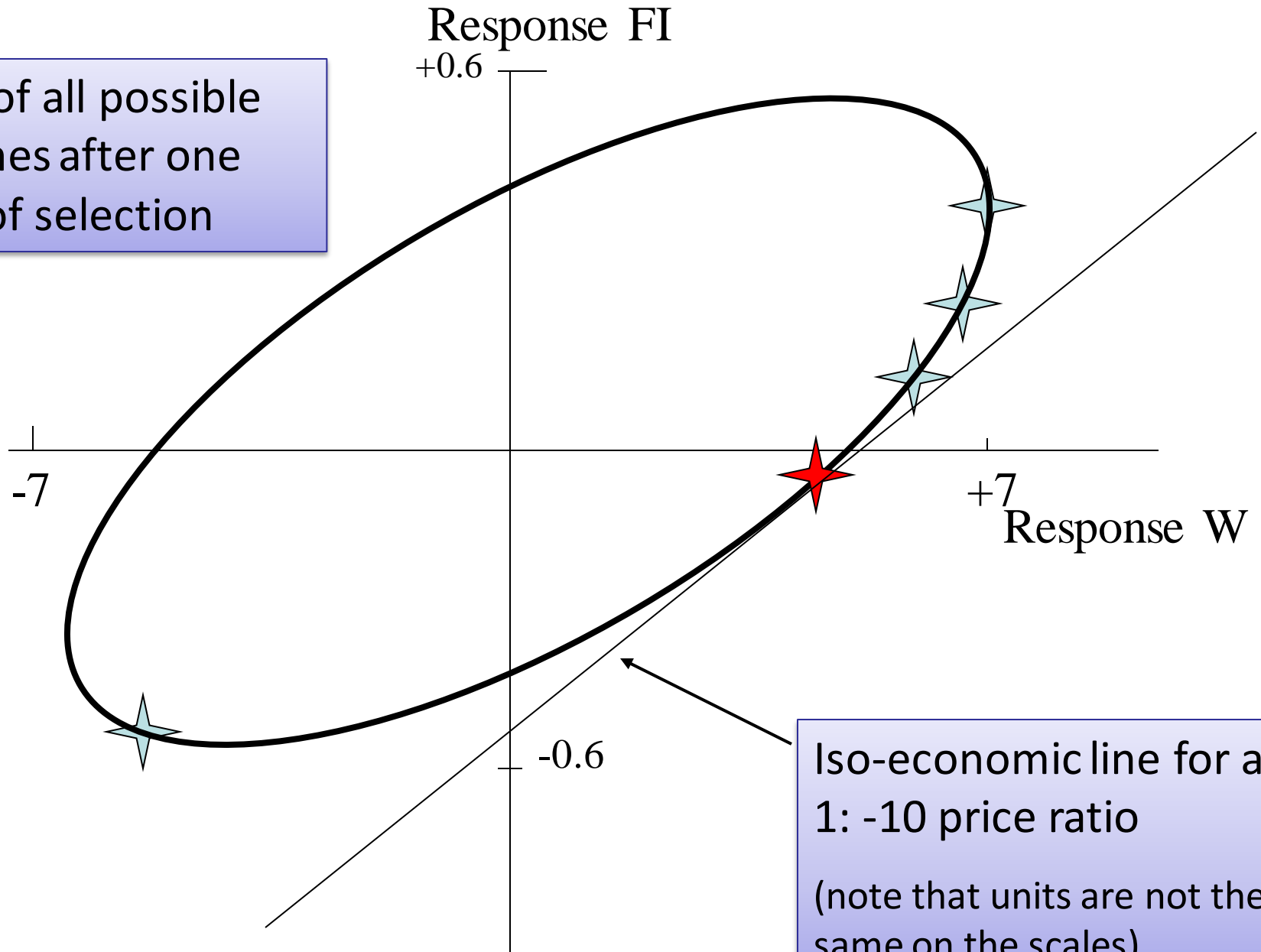
But at the expense of less improvement in weight

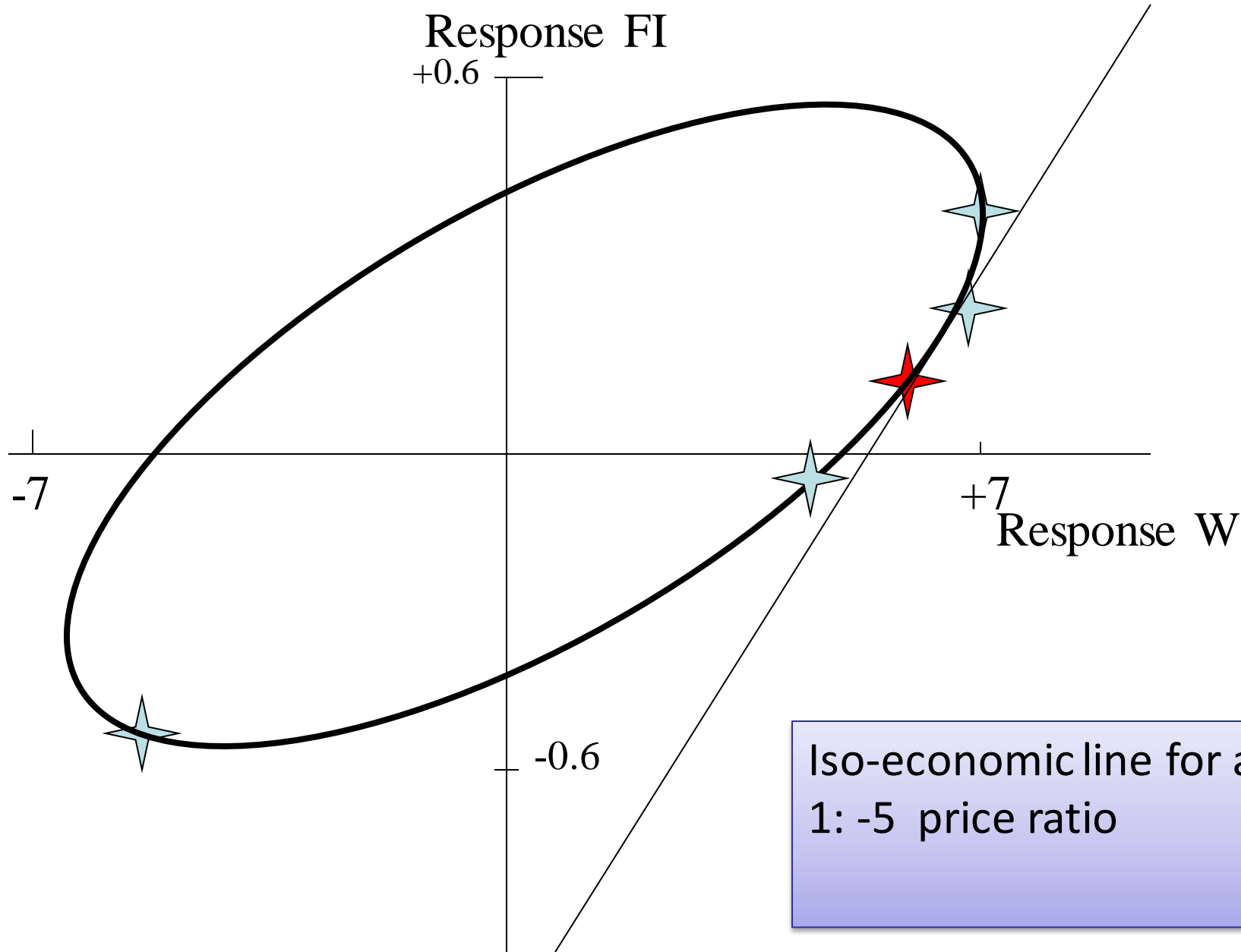
And the response is that future animals will eat less

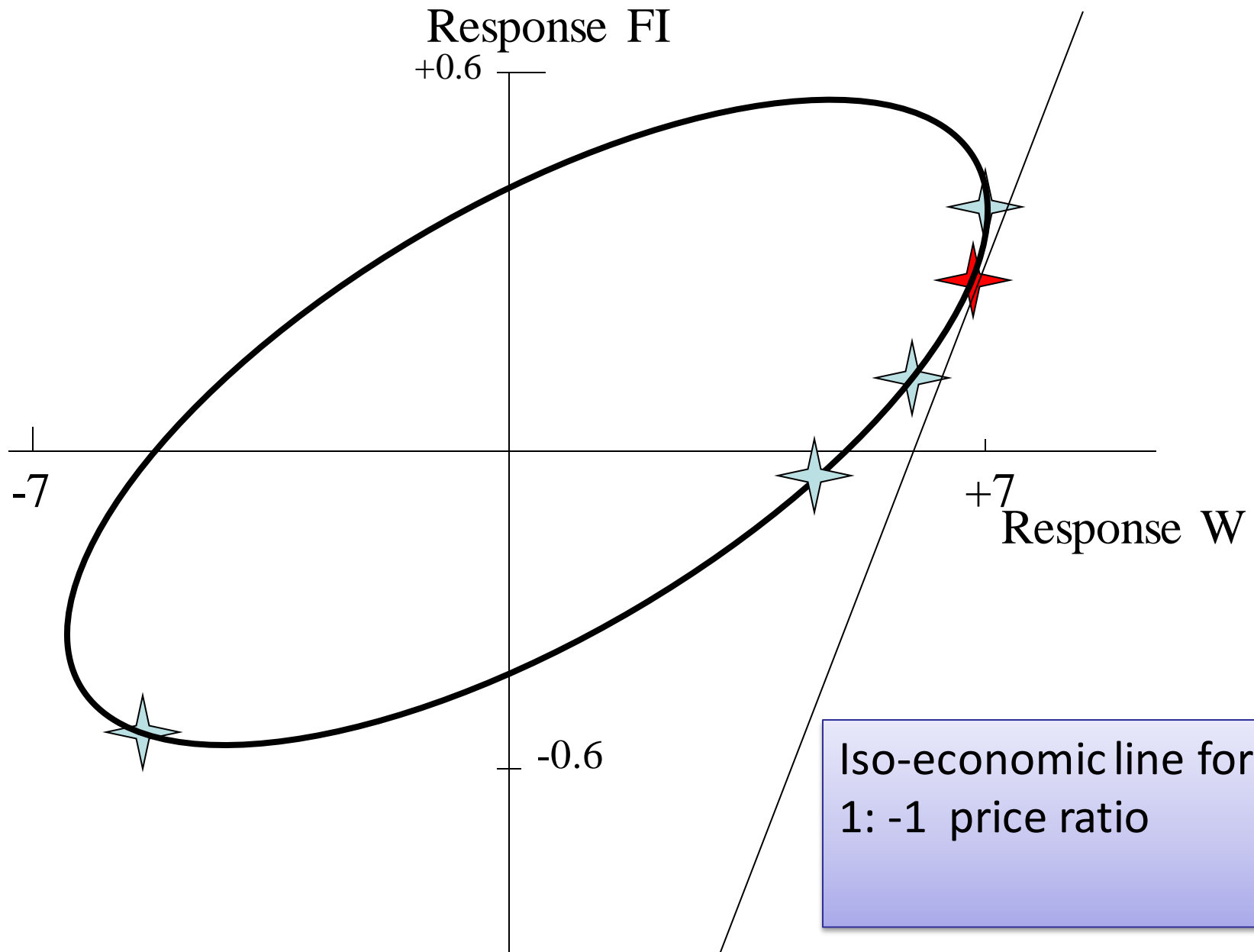


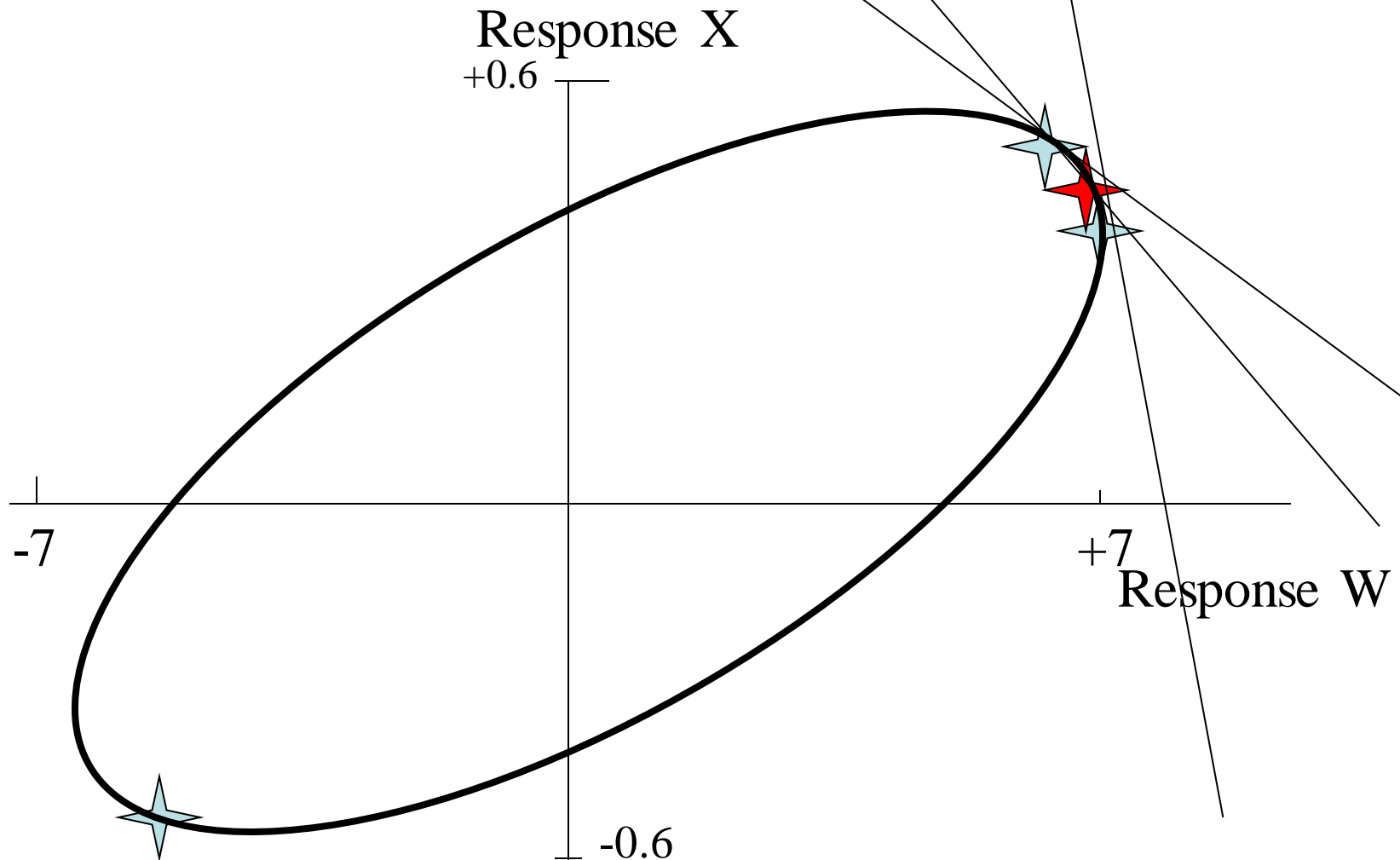
Select on body weight + feed intake Objective is $1.0 \cdot BW - 4 \cdot FI$

Ellipse of all possible outcomes after one round of selection









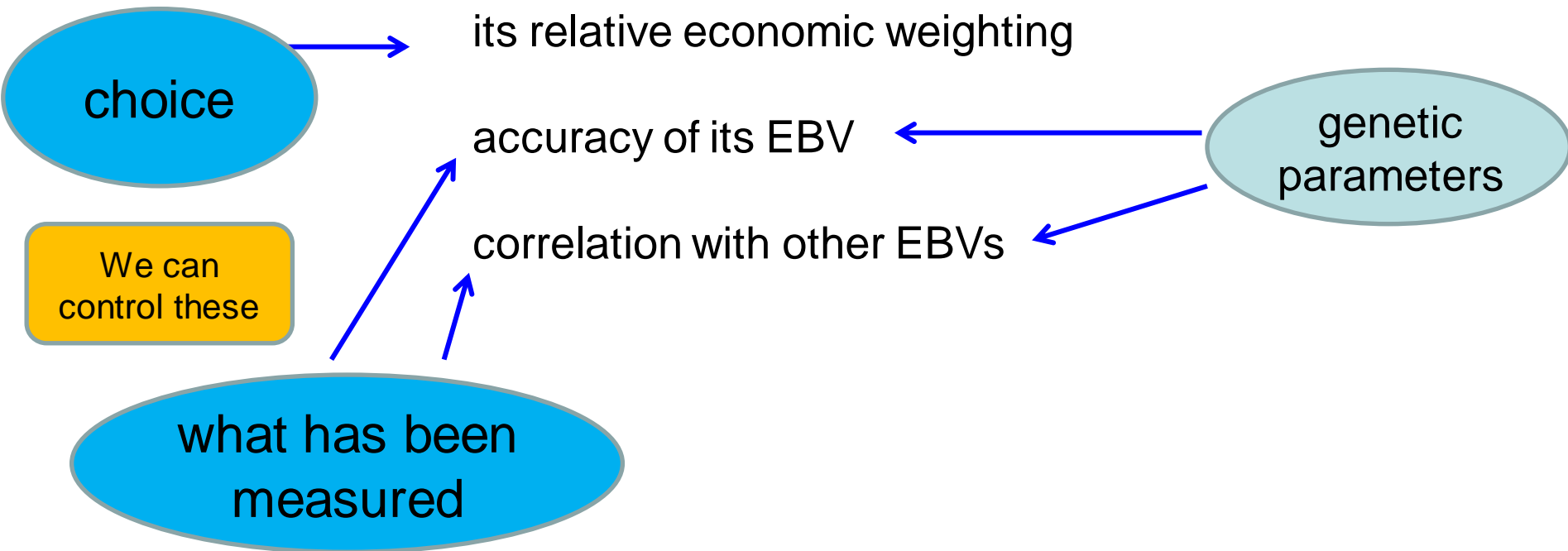
This is a case where both economic values are positive

The genetic correlation is now favourable

Note at the direction of selection is quit insensitive to changes in economic values

Some important points about MT selection

1 The ultimate response of a trait will depend on:



Some important points

The EBV of a trait can reflect another trait

Body Weight X_{11}

Body Weight on relatives X_{12}

EBV for fertility (g_2)

In this case, selection of $EBV_{\text{fertility}}$ will increase fertility,
but it will even more increase body weight!

Can predict changes from MT selection using selection index

Need to understand some important MT principles → ellipse

Some important points

The index weight is not always reflecting the response

Weight can be positive, response can be negative

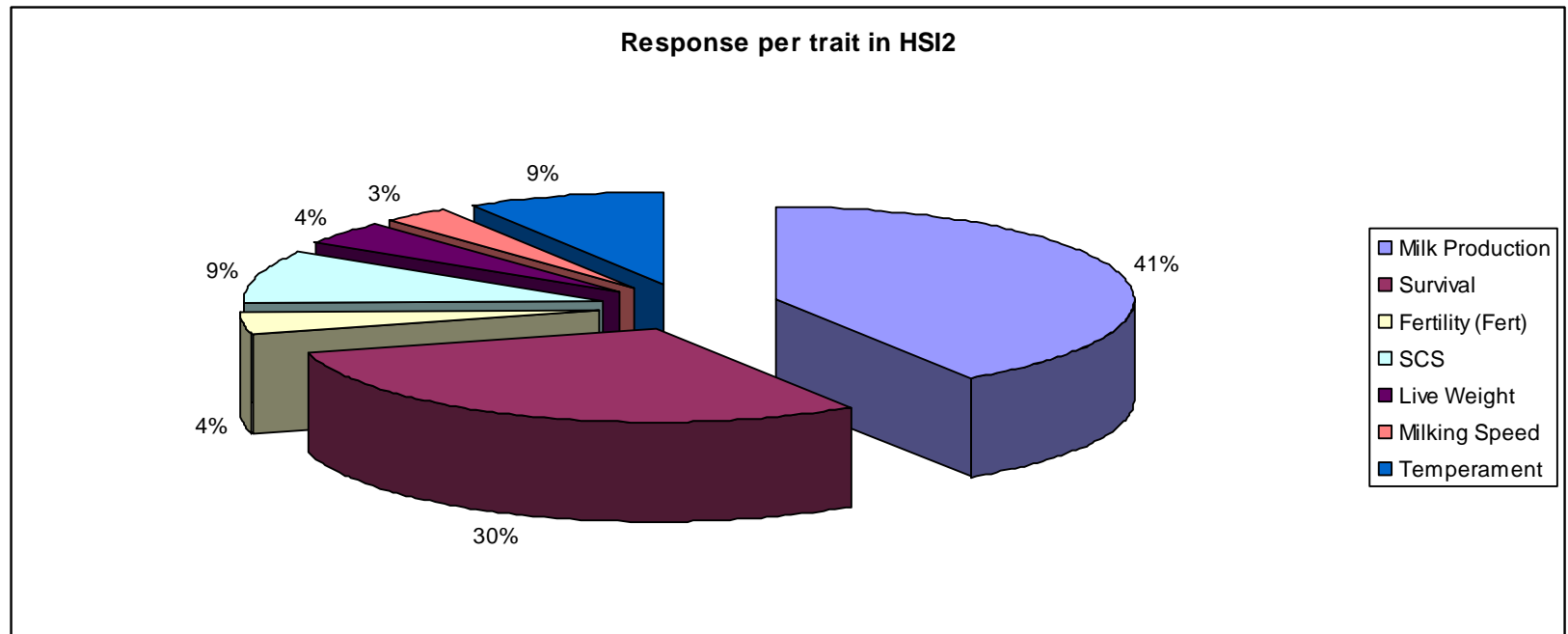
The weight does not always reflect the economic value

Econ Value can be negative, weight can be positive

Some traits are easier to improve than others

Some traits are easier to improve jointly than others

What is this “percent emphasis”



e.g. UK: “55% focus on fitness traits” ; “remaining 45% of index covering production”

Index weights?

Selection response?

Index weight vs Selection response

	<u>weights</u>	<u>response</u>	<u>weights</u>	<u>response</u>
Milk Kg	0	321	1	368
Protein Kg	1	11.0	0	9.6



provided by Hoard's Dairyman

	<u>weights</u>	<u>response</u>	<u>weights</u>	<u>response</u>
Protein Kg	4	9.0	4	11.0
Live Weight Kg	-1	12.5	0	24.9

Case study 1

Dairy: select bulls (50 prog) on

- milk production
- feed intake



provided by Hoard's Dairyman

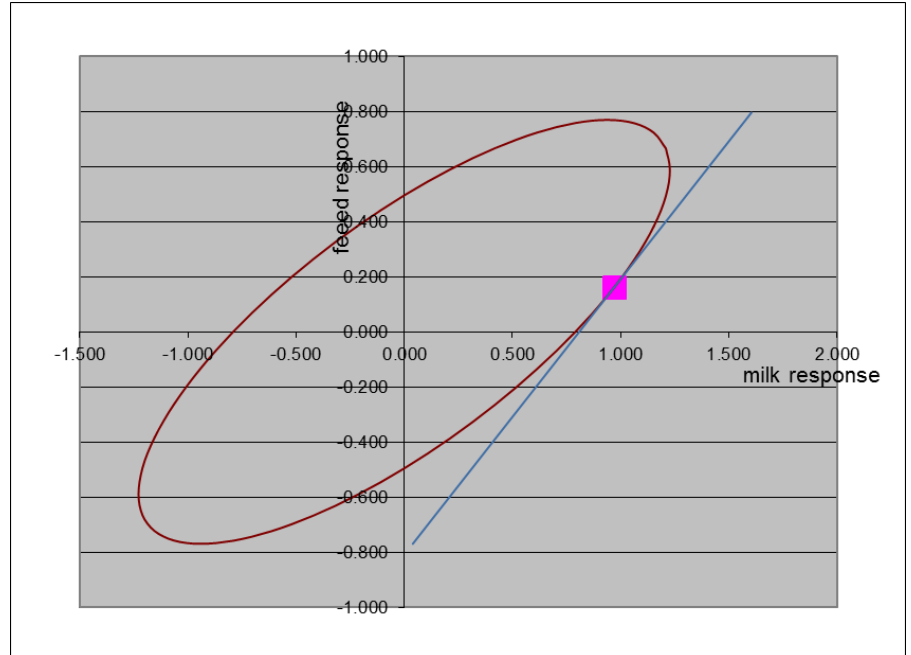
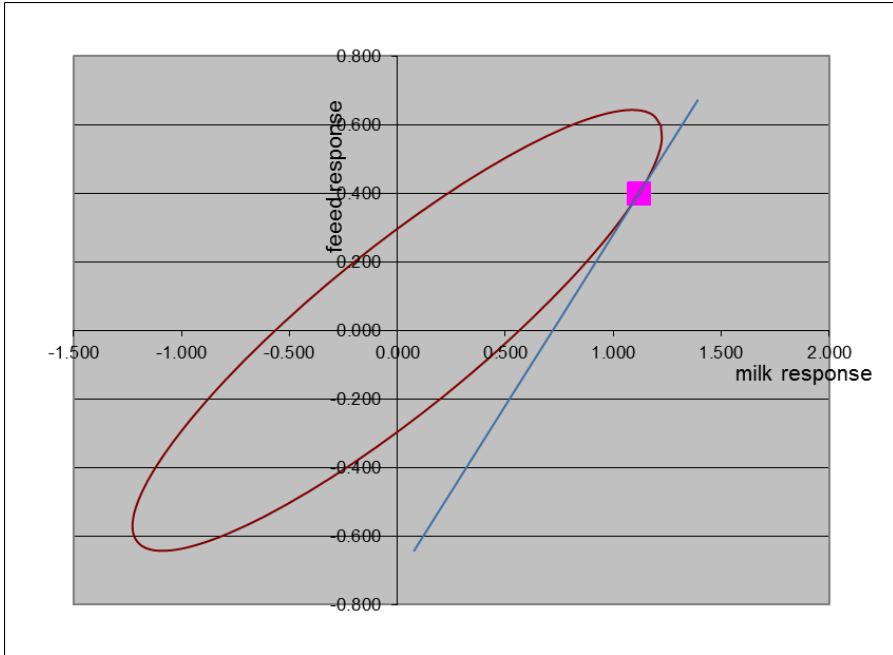
	μ	σ_p	h^2
Milk (kg/day)	25	2.5	.3
Feed intake (kg/day)	20	2.0	.2
	$r_g = .70$	$r_p = 0.40$	

Selection for Milk Yield and Feed Intake

<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>
0.2	0	50	-	1.23	0.56
0.2	0	50	50	1.23	0.59
0.2	-0.2	50	-	1.23	0.56
0.2	-0.2	50	50	0.97	0.16
0.2	-0.3	50	-	1.23	0.56
0.2	-0.3	50	50	0.52	-0.20
0.2	-0.3	50	10	0.79	0.14

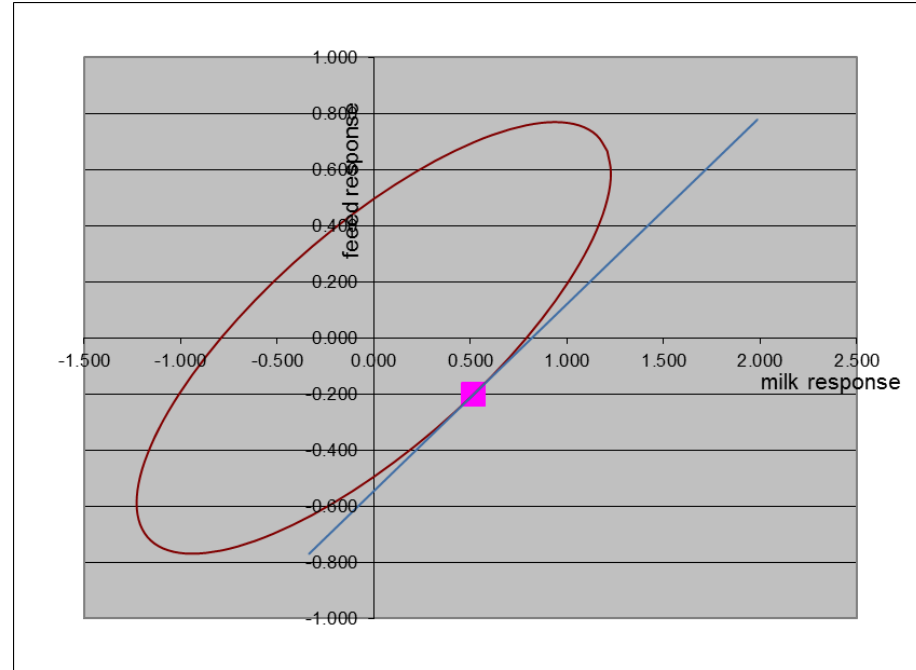
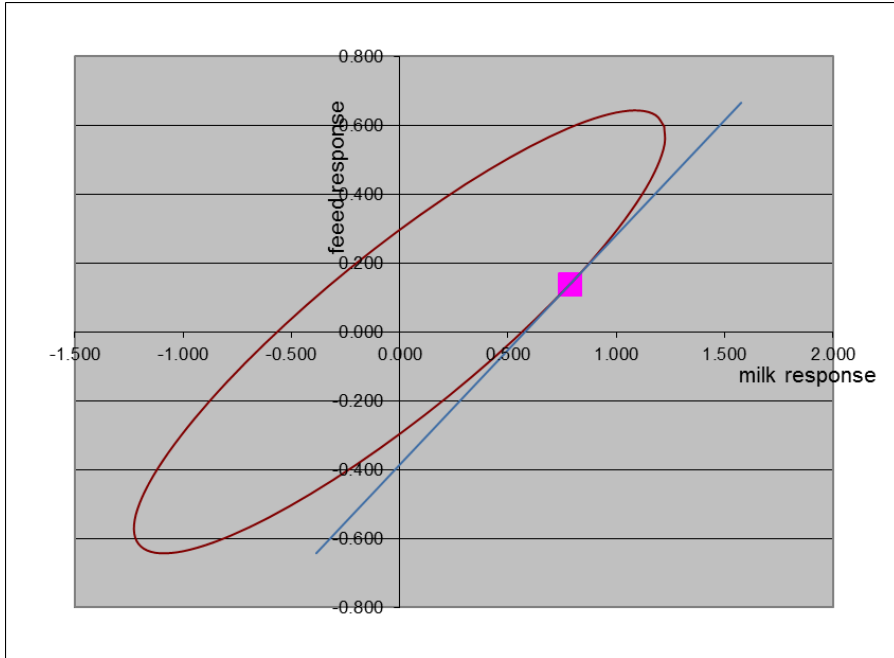
To achieve response for a trait, we need to give it some weight
but we also need some data!

Selection for Milk Yield and Feed Intake



	<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>
left	0.2	-0.2	50	10	1.23	0.56
right	0.2	-0.2	50	50	0.97	0.16

Selection for Milk Yield and Feed Intake



	<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>
left	0.2	-0.3	50	10	0.79	0.14
right	0.2	-0.3	50	50	0.52	-0.20

Case study 2

Dairy: select bulls (50 prog) on

- milk production
- fertility



provided by Hoard's Dairyman

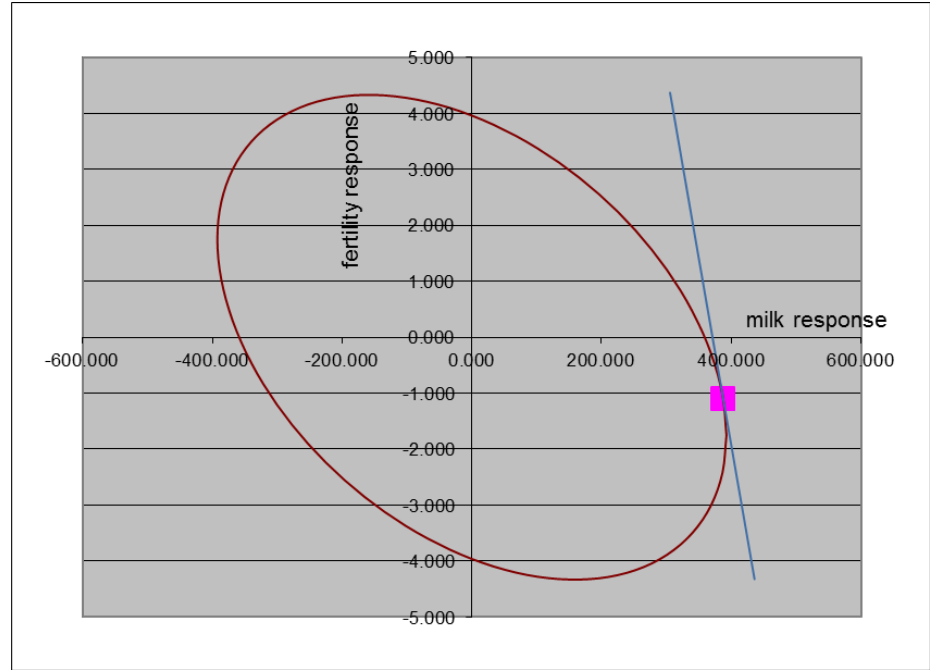
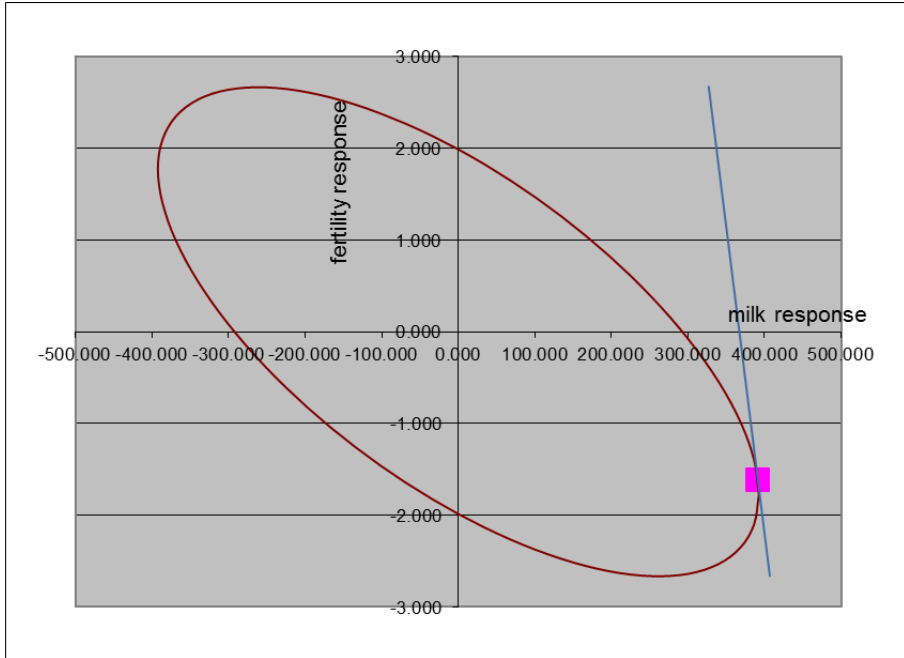
	μ	σ_p	h^2
Milk (kg/lac)	8,000	800	.3
Fertility(%)	70	46	.03
	$r_g = -.25$	$r_p = -0.1$	

Selection for milk Yield and Fertility

<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
<i>milk</i>	<i>fertility</i>	<i>milk</i>	<i>fertility</i>	<i>milk</i>	<i>fertility</i>
0.2	0	50	-	392	-1.78
0.2	0	50	50	392	-1.75
0.2	3	50	-	392	-1.78
0.2	3	50	50	387	-1.09
0.2	8	50	-	392	-1.78
0.2	8	50	50	352	0.17
0.2	8	50	10	381	-1.25

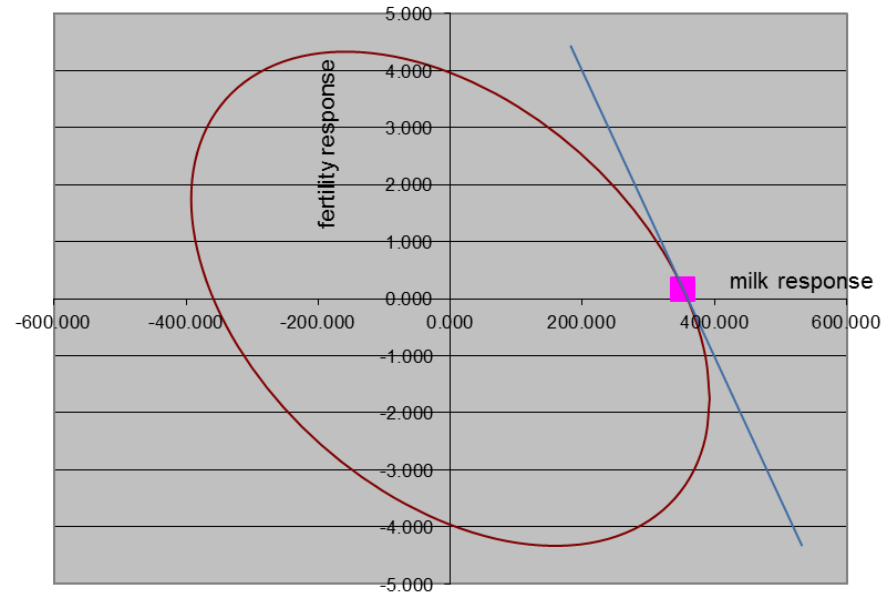
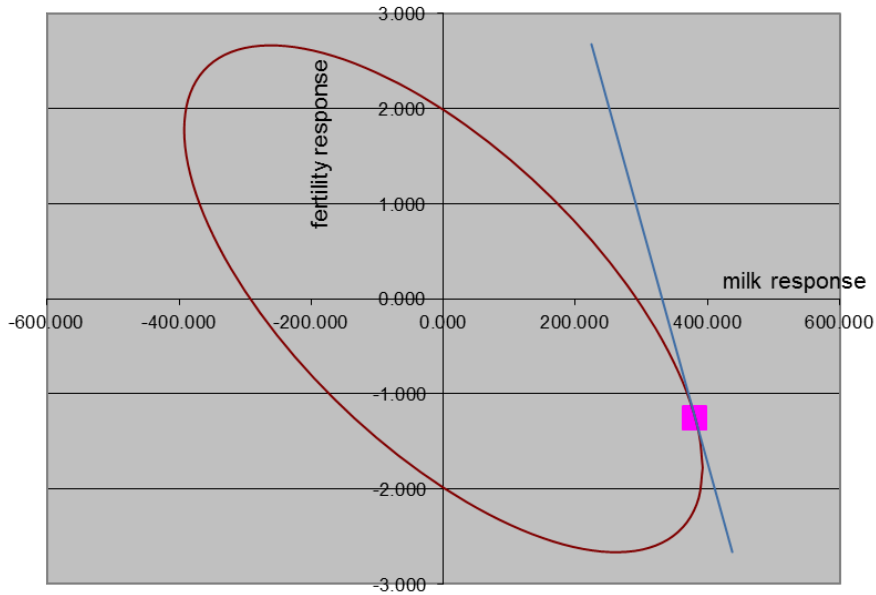
To achieve response for a trait, we need to give it some weight
but we also need some data!

Selection for milk Yield and Fertility



	<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
	<i>milk</i>	<i>fertility</i>	<i>milk</i>	<i>fertility</i>	<i>milk</i>	<i>fertility</i>
left	0.2	3	50	10	391	-1.61
right	0.2	3	50	50	387	-1.09

Selection for milk Yield and Fertility

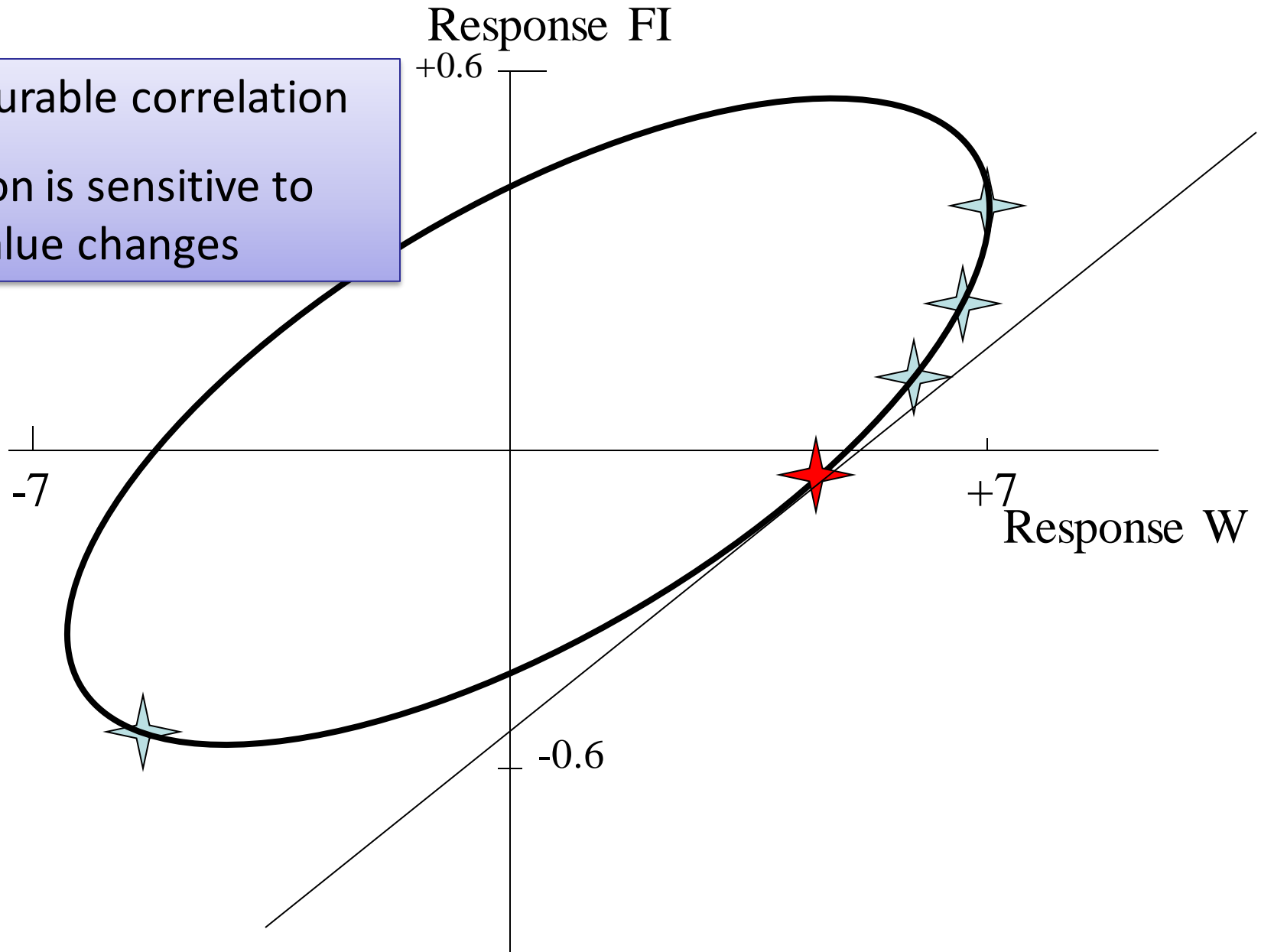


	<i>economic</i>	<i>weights</i>	<i>progeny</i>	<i>measured</i>	<i>response</i>	<i>(4 yrs)</i>
	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>	<i>milk</i>	<i>feed</i>
left	0.2	8	50	10	381	-1.25
right	0.2	8	50	50	352	0.17

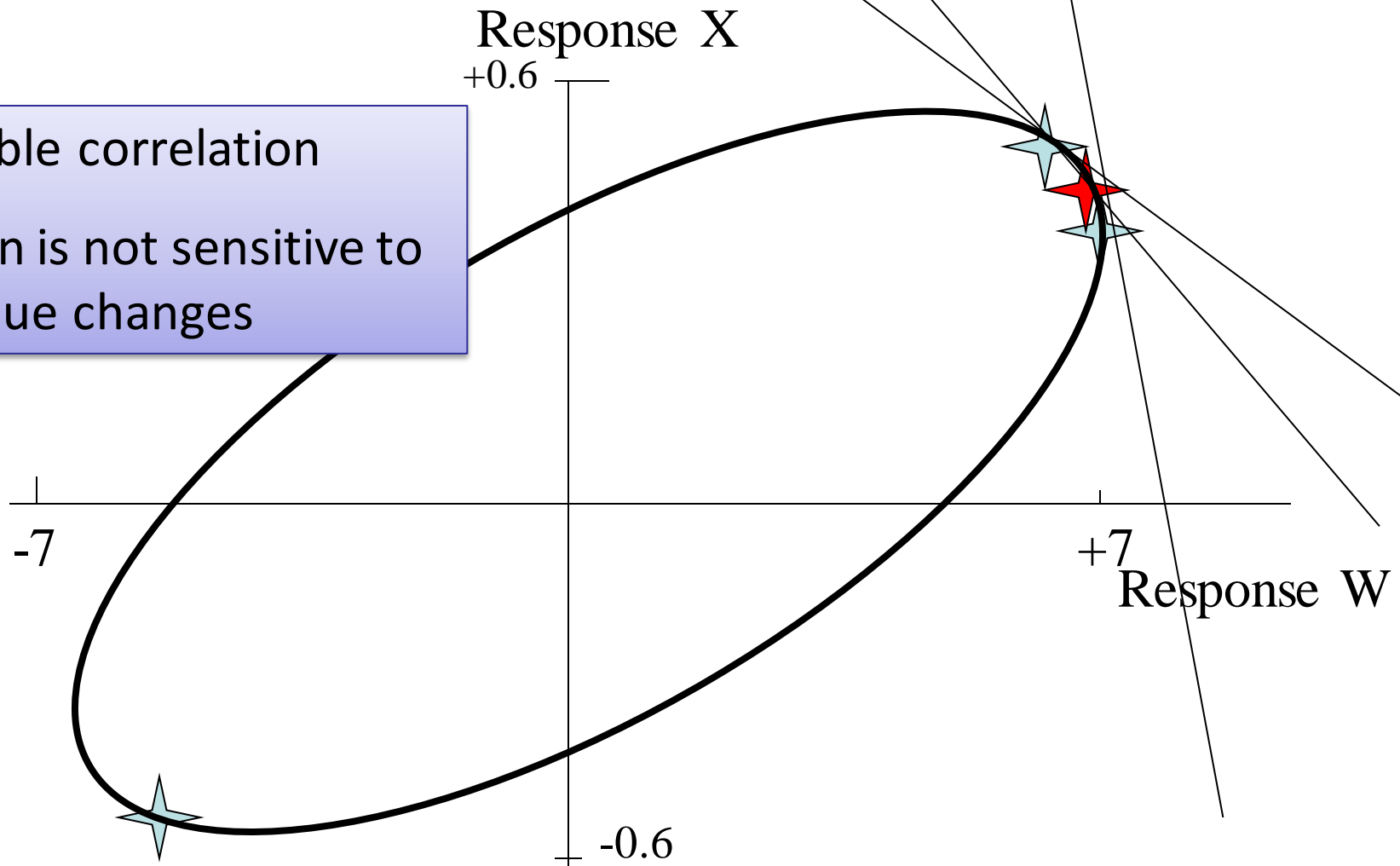
A challenge

- Assume two traits have a positive economic values
- Why is selection for these traits less sensitive to economic values when they are positively correlated compared to when they are negatively correlated

Unfavourable correlation
Direction is sensitive to
econ value changes



Favourable correlation
Direction is not sensitive to
econ value changes

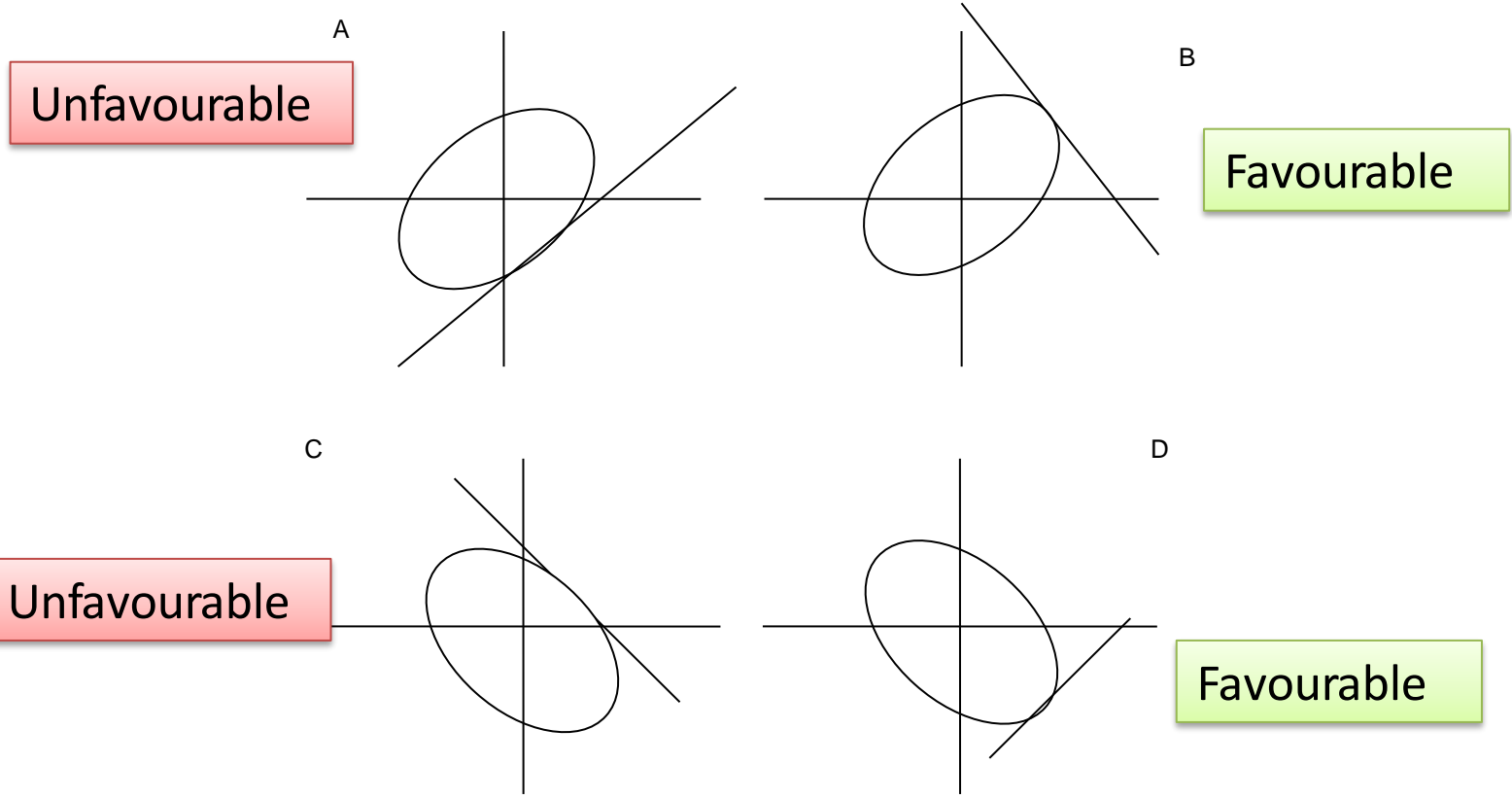


This is a case where both economic values are positive

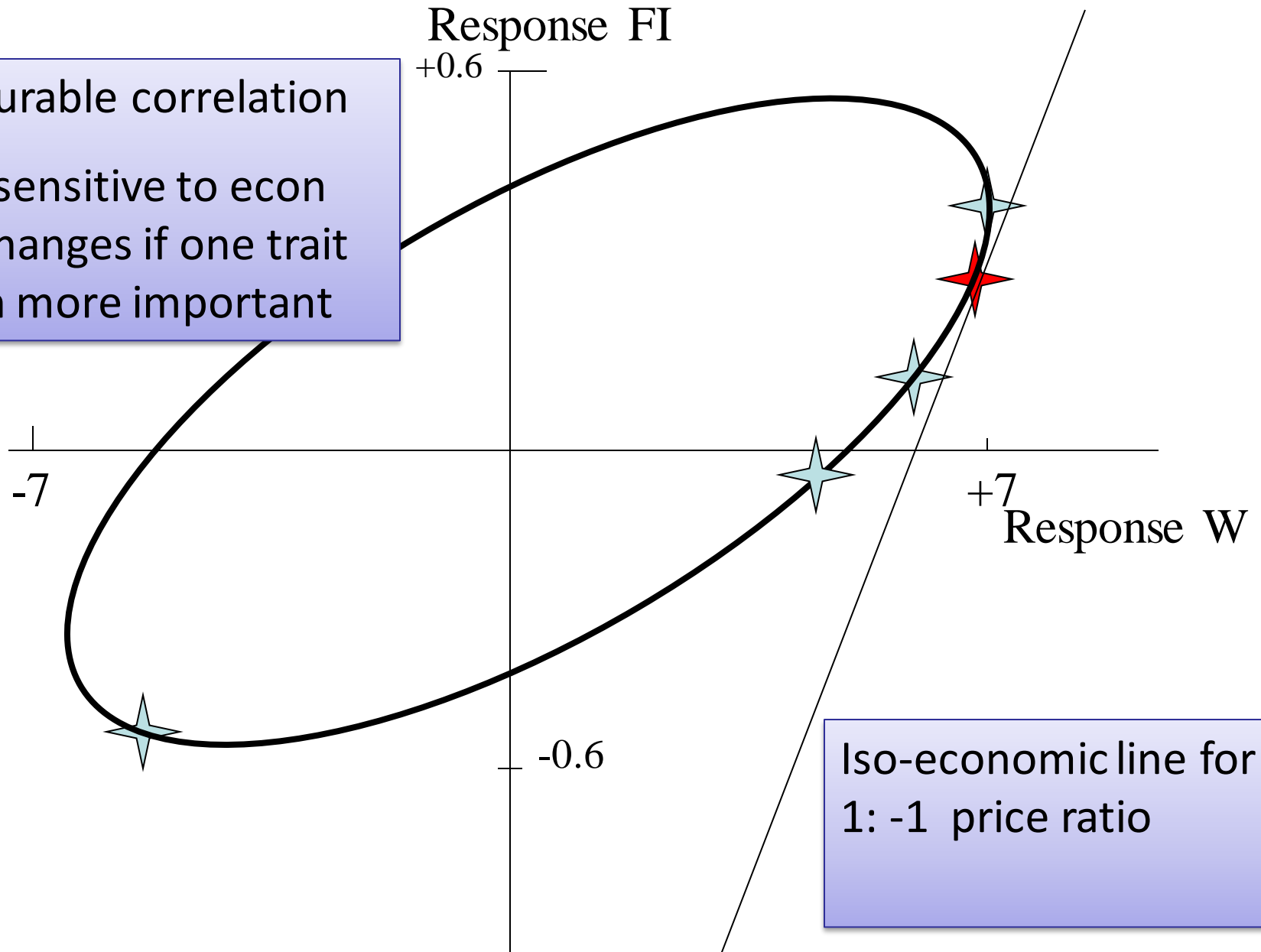
The genetic correlation is now favourable

Note at the direction of selection is quit insensitive to changes in economic values

Correlation	Sign of economic weights	
	Equal	Opposite
Positive	Favourable (B)	Unfavourable (A)
Negative	Unfavourable (C)	Favourable (D)



Unfavourable correlation
Not so sensitive to econ
value changes if one trait
is much more important



Selection index with 'desired gains'

- Rather than
 - determine econ. values >>>> response
 - We desire a response >>>> economic values
(implicit)

When useful?

Using EBVs as Selection Criteria

- 1) Define the overall objective (e.g. profit per animal).
- 2) Develop a linear breeding goal: $H = v_1g_1 + v_2g_2 \dots v_n g_n = \mathbf{v}'\mathbf{g}$
- 3) Derive the economic value (v) for each trait in H
- 3) Derive a linear index (I) of information sources that maximizes the accuracy of the index with H : $\mathbf{b} = \mathbf{P}^{-1}\mathbf{G}\mathbf{v}$
 - a. Based on phenotypes: $I = b_1x_1 + b_2x_2 \dots b_mx_m = \mathbf{b}_x'\mathbf{x}$
 - a. Based on EBV: $I = b_1\hat{g}_1 + b_2\hat{g}_2 \dots b_m\hat{g}_m = \mathbf{b}_{EBV}'\hat{\mathbf{g}}$

**Maximize
Response in
Overall
Objective**

**Optimize
Response in
Individual
Traits**

Slides from Jack Dekkers, WCGALP 2014, Vancouver

Selection Index Theory

$$H = v_1 g_1 + v_2 g_2 \dots v_n g_n = \mathbf{v}'\mathbf{g}$$

$$I = b_1 \hat{g}_1 + b_2 \hat{g}_2 \dots b_m \hat{g}_m = \mathbf{b}_{EBV}' \hat{\mathbf{g}}$$

Optimal Index Weights (multi-trait EBV): $\mathbf{b} = \mathbf{P}^{-1}\mathbf{G}\mathbf{v}$

- if traits in $I =$ traits in H : $\mathbf{b}_{EBV} = \mathbf{v}$
- if traits in $I \neq$ traits in H : $\mathbf{b}_{EBV} = \mathbf{b}_{g_H g_I}' \mathbf{v}$ function of genetic parameters

(Schneeberger et al. 1992)

Responses to selection:

- in individual traits: $\mathbf{S}_g = [S_{g_1}, S_{g_2}, \dots, S_{g_m}] = \mathbf{b}_{g_H g_I}' \mathbf{b}_{EBV}' \mathbf{G}_{EBV} / \sigma_I \stackrel{\text{traits } I = H}{=} \mathbf{v}' \mathbf{G}_{EBV} / \sigma_I$

with $\sigma_I = \mathbf{b}_{EBV}' \mathbf{G}_{EBV} \mathbf{b}_{EBV}$

Trait responses depend on:

- Genetic parameters, economic values
- Variance-covariance matrix of EBV $= \mathbf{G}_{EBV} =$

$$\begin{bmatrix} r_1^2 \sigma_{g_1}^2 & & & \\ & r_2^2 \sigma_{g_2}^2 & & \\ & & & \\ & & & r_m^2 \sigma_{g_m}^2 \end{bmatrix}$$

EBV covariances
genet.param.
- Information
- phenotypes
- genomics

\mathbf{G}_{EBV} captures the impact of alternate breeding program and phenotyping designs on trait responses – ‘artificial evolution’ (Gibson 1989)

Lost Responses from Uncertainty about Economic Values

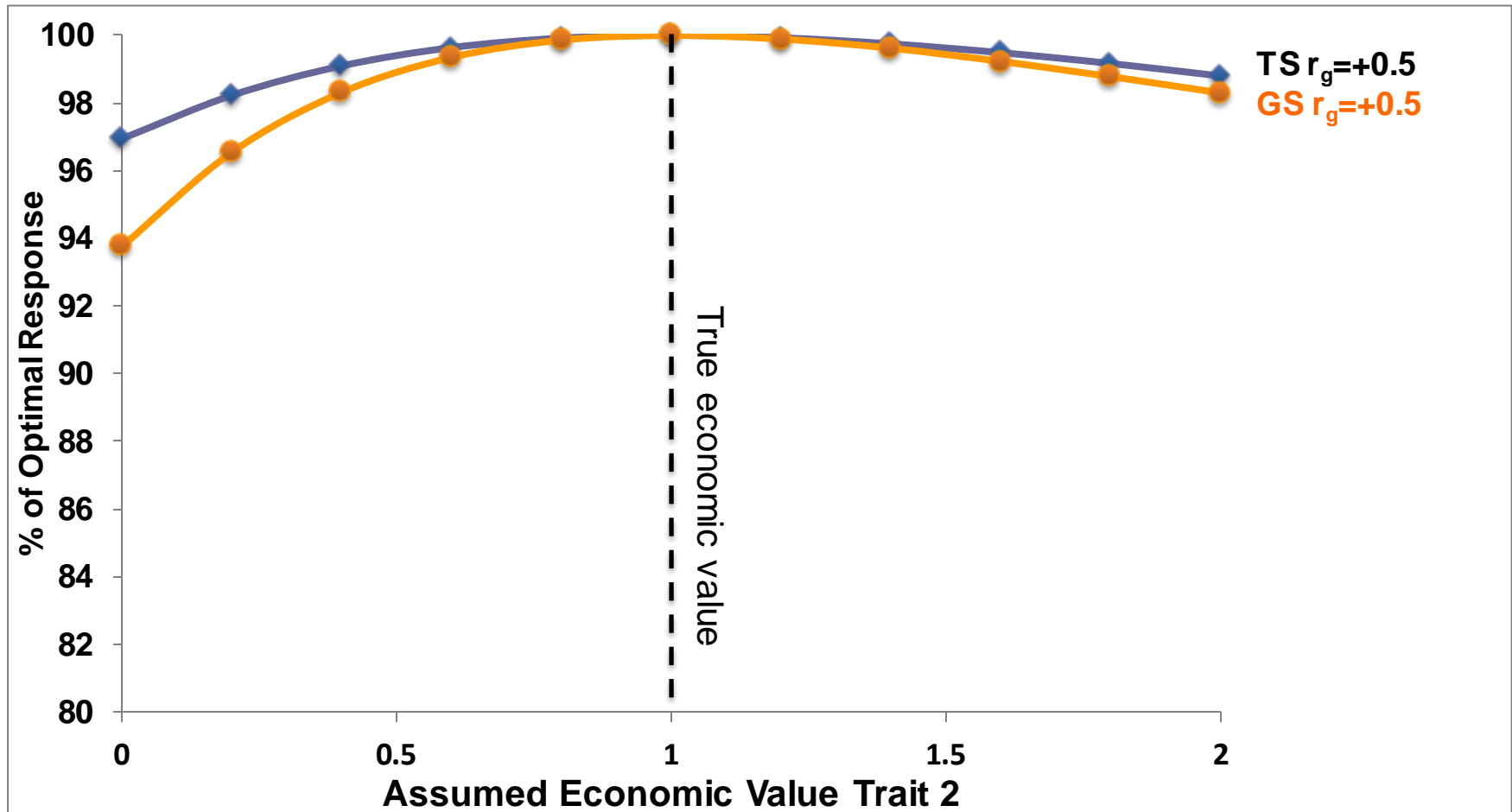
TS = traditional selection
GS = genomic selection

$r_1 = 0.64$

$r_2 = 0.43$

$r_1 = 0.90$

$r_2 = 0.90$



True economic values = 1 for both traits

Lost Responses from Uncertainty about Economic Values

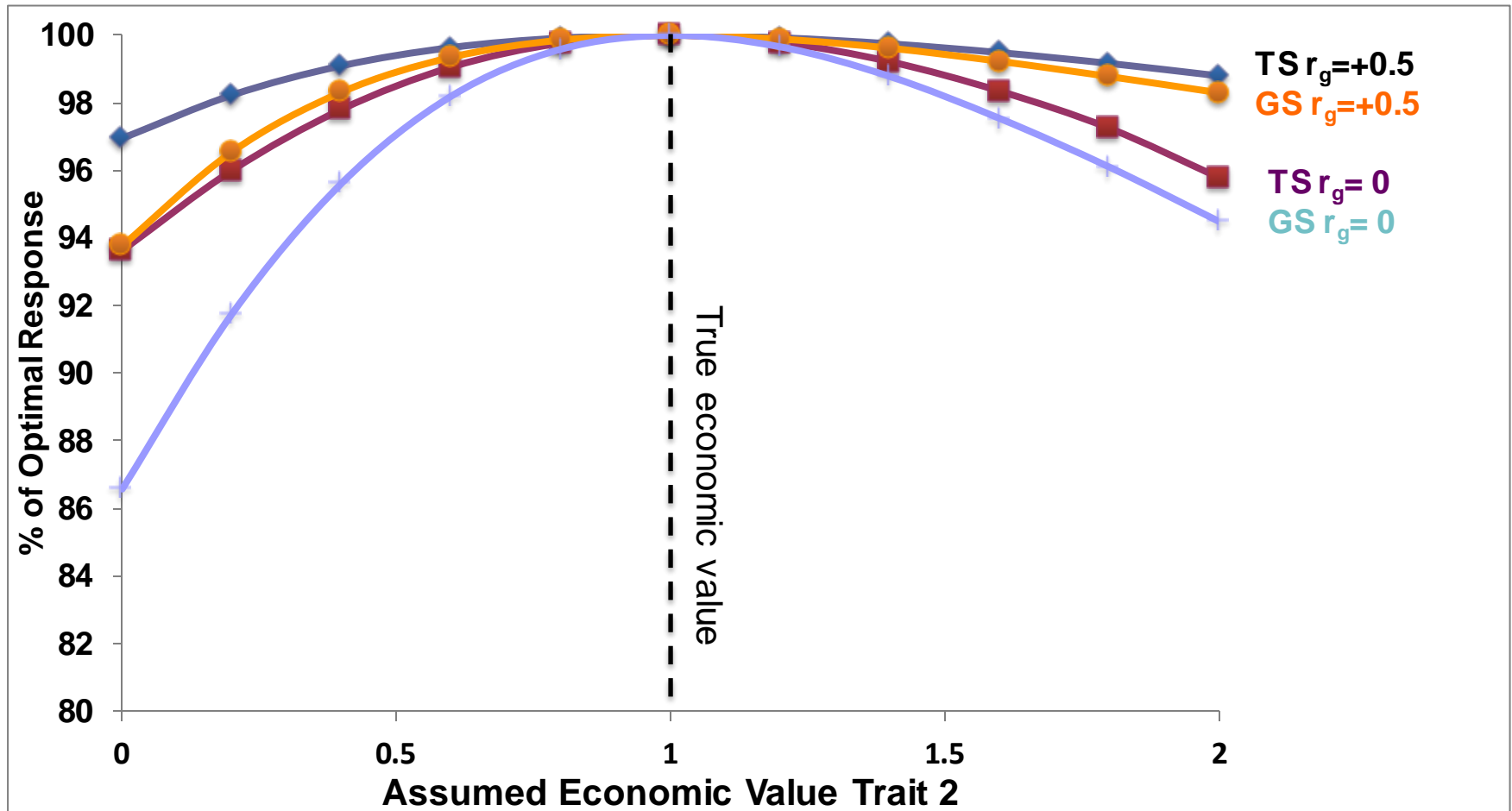
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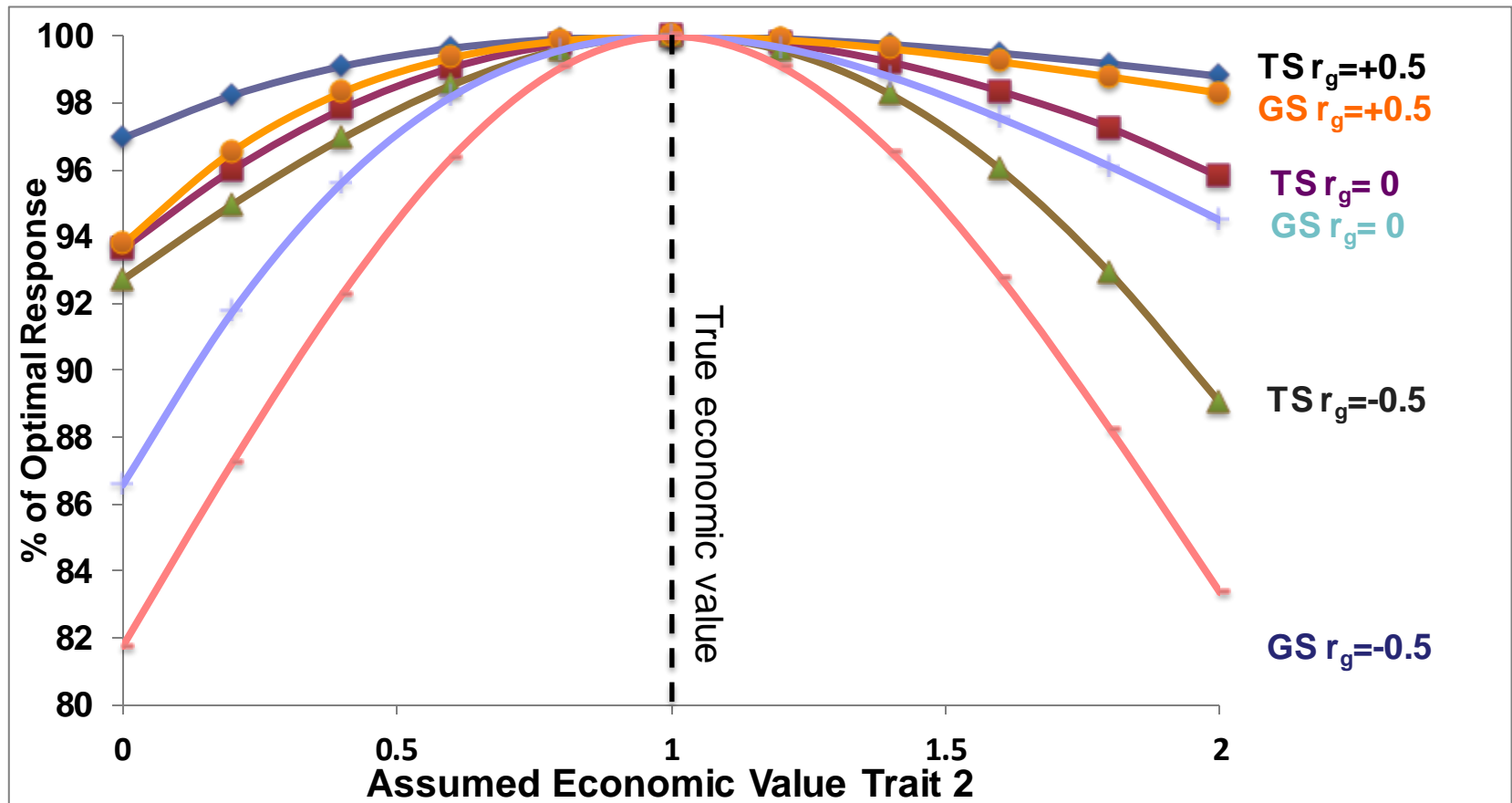
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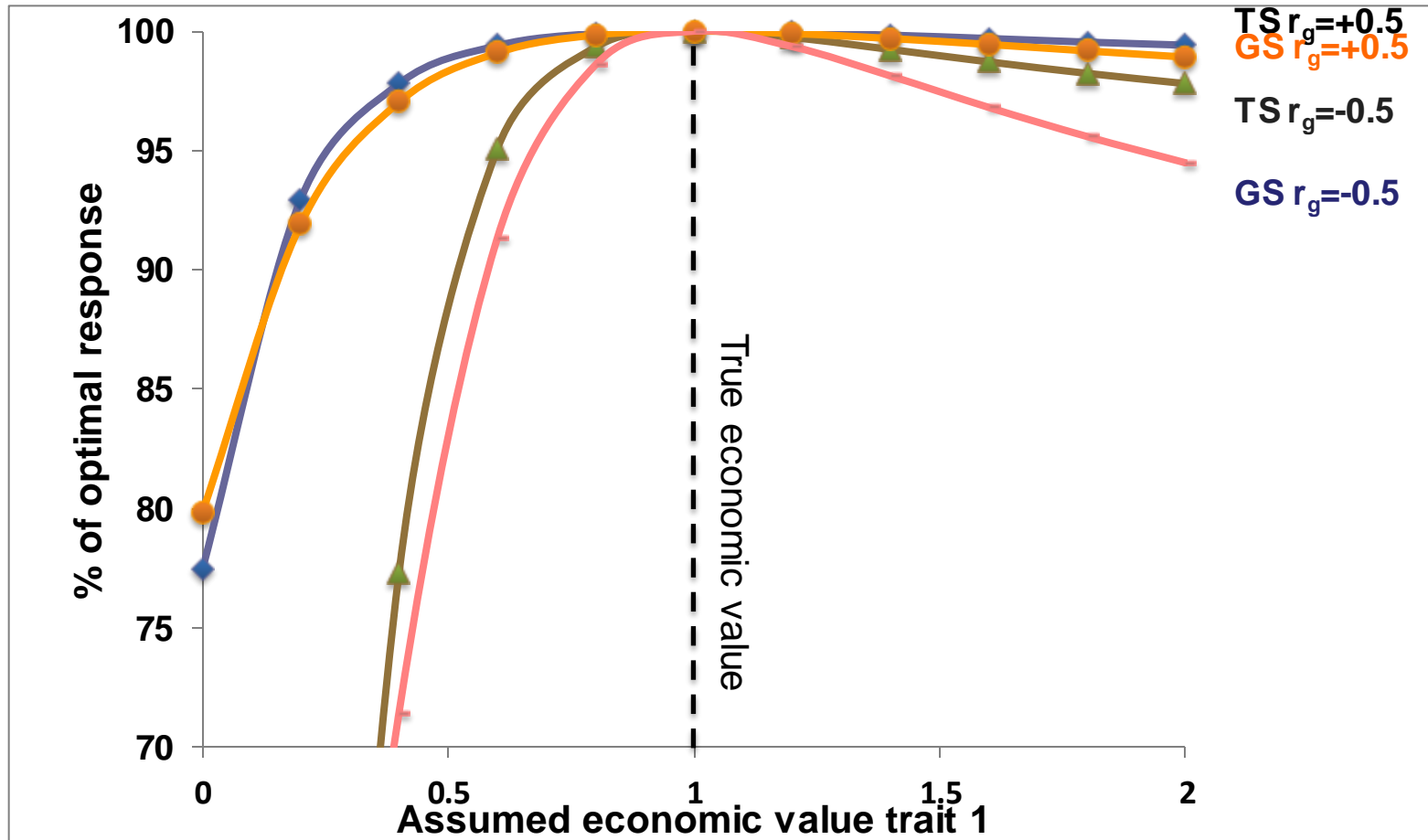
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True economic values = 1 for both traits

Breeding Goals and Phenotyping Programs for Multi-Trait Improvement in the Genomics Era

1) Impact of errors in economic values with genomic selection


- Importance of having accurate economic values is greater with genomic selection**
- Impact of using suboptimal indexes is greater with genomic selection**

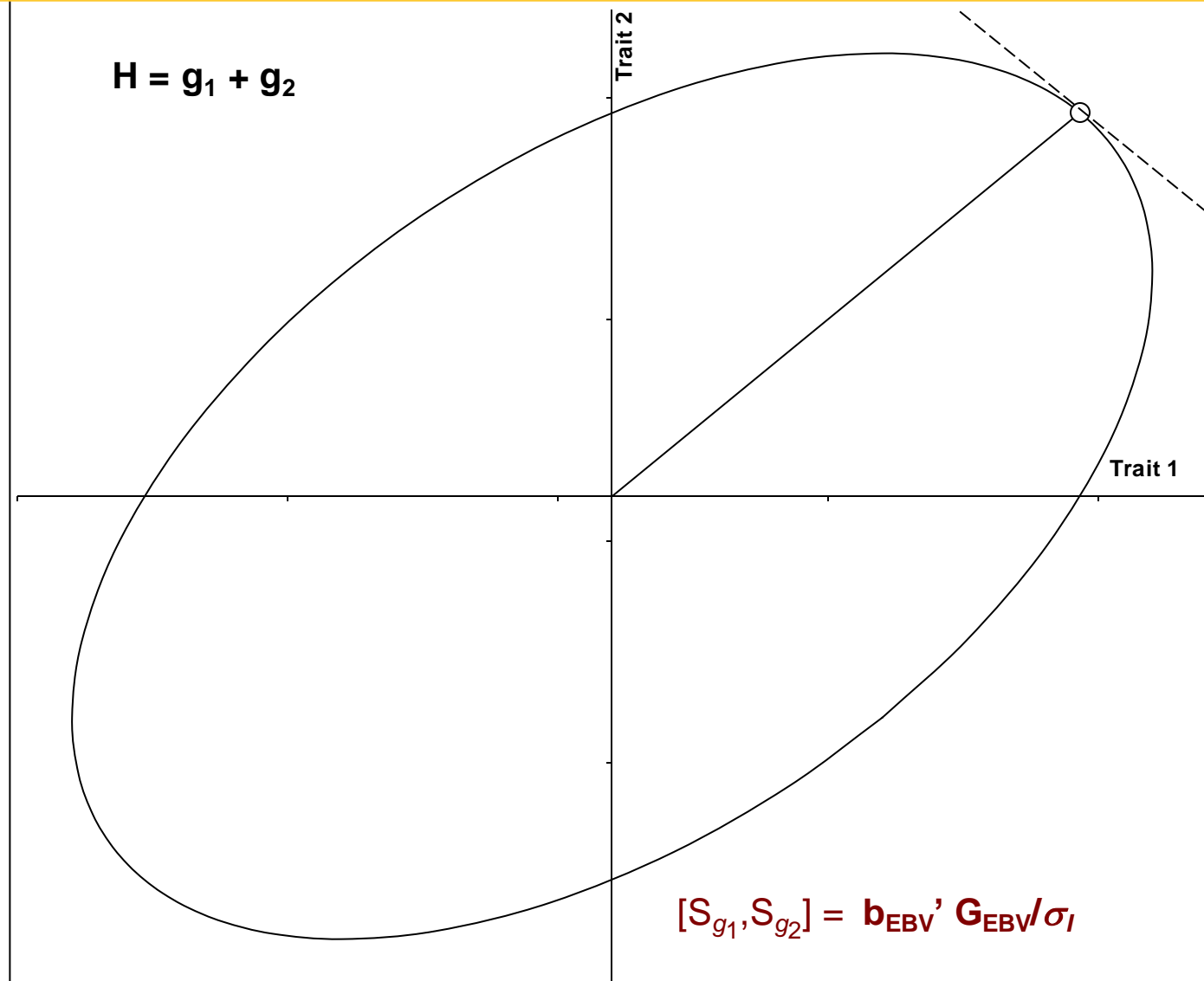
Use of Genomics to Shape Artificial Evolution

$$v_1 = v_2 = 1$$

$$\sigma_{g_1} = \sigma_{g_2}$$

$$r_g = r_p = 0.5$$

	Trait 1	Trait 2
r_{EBV} TS	0.55	0.22
r_{EBV} GS	0.75	0.75
	TBV	TBV





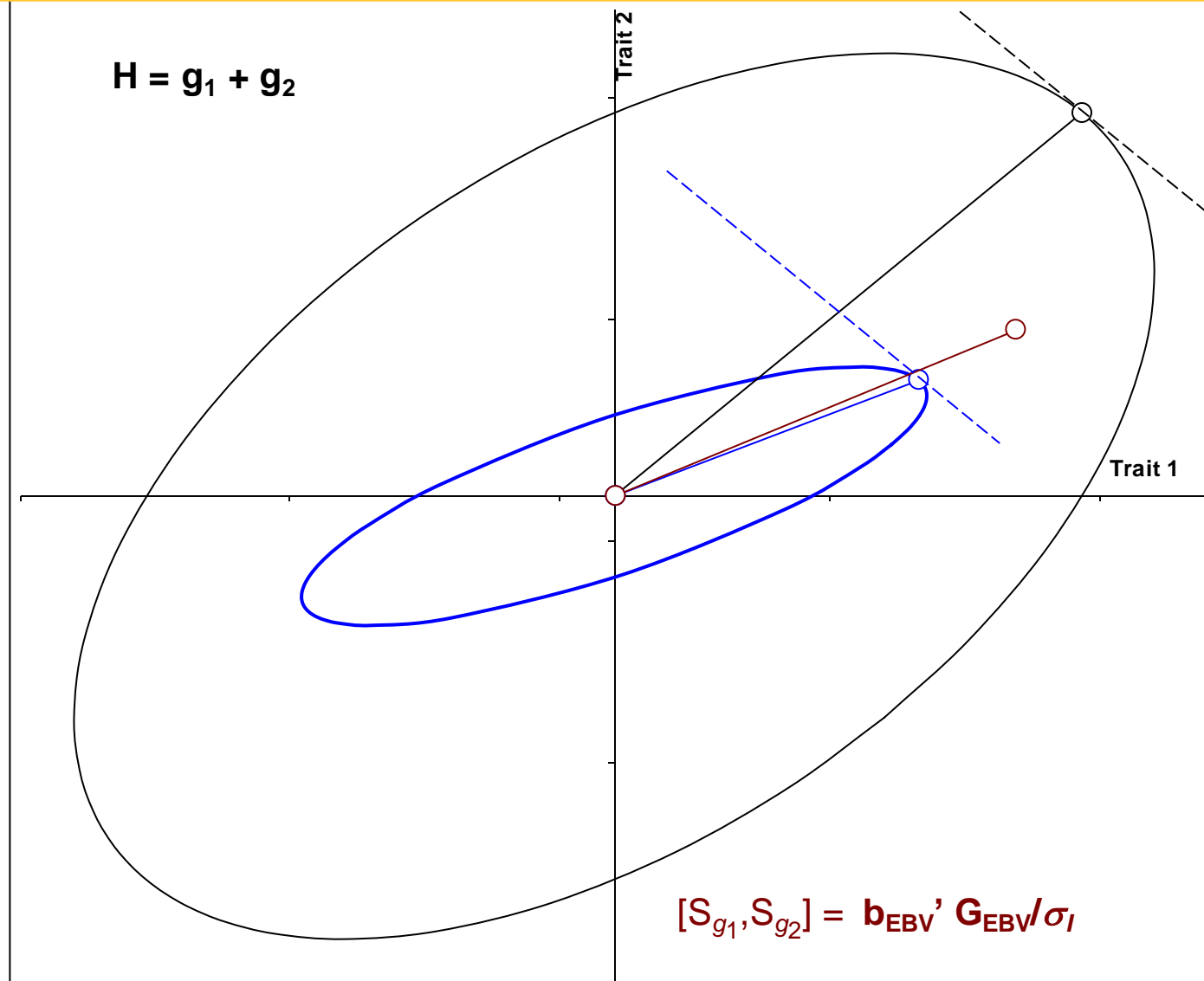
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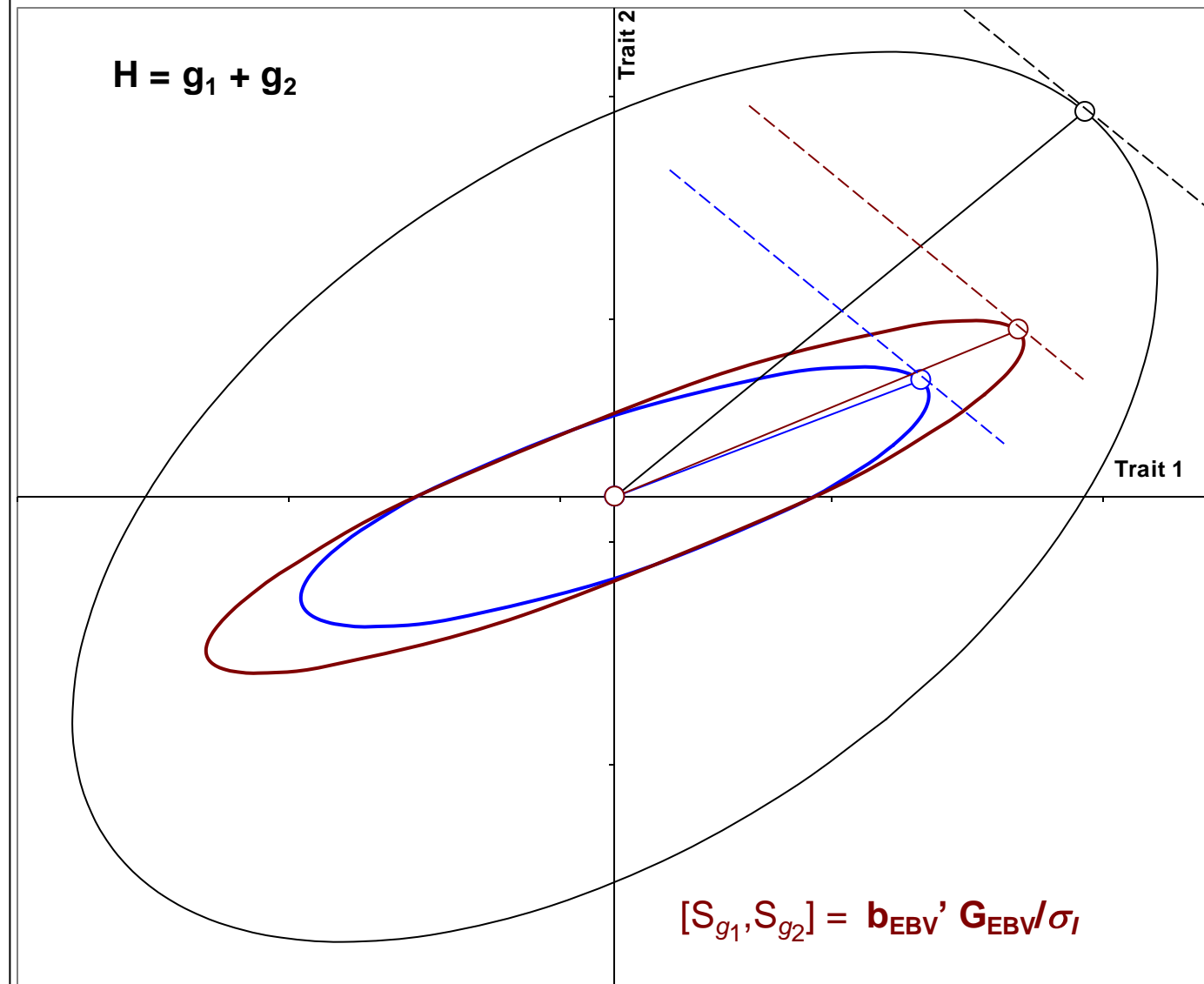
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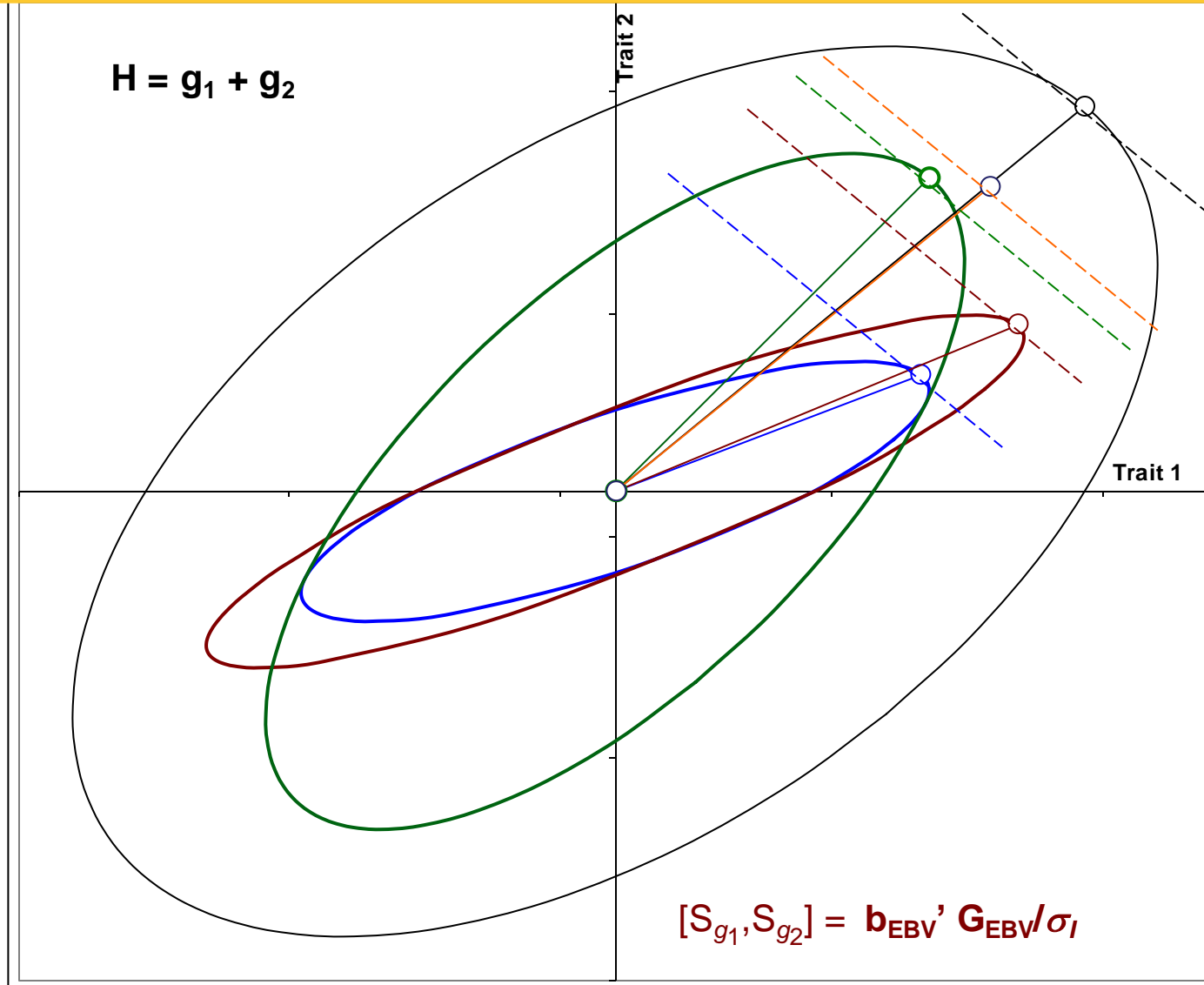
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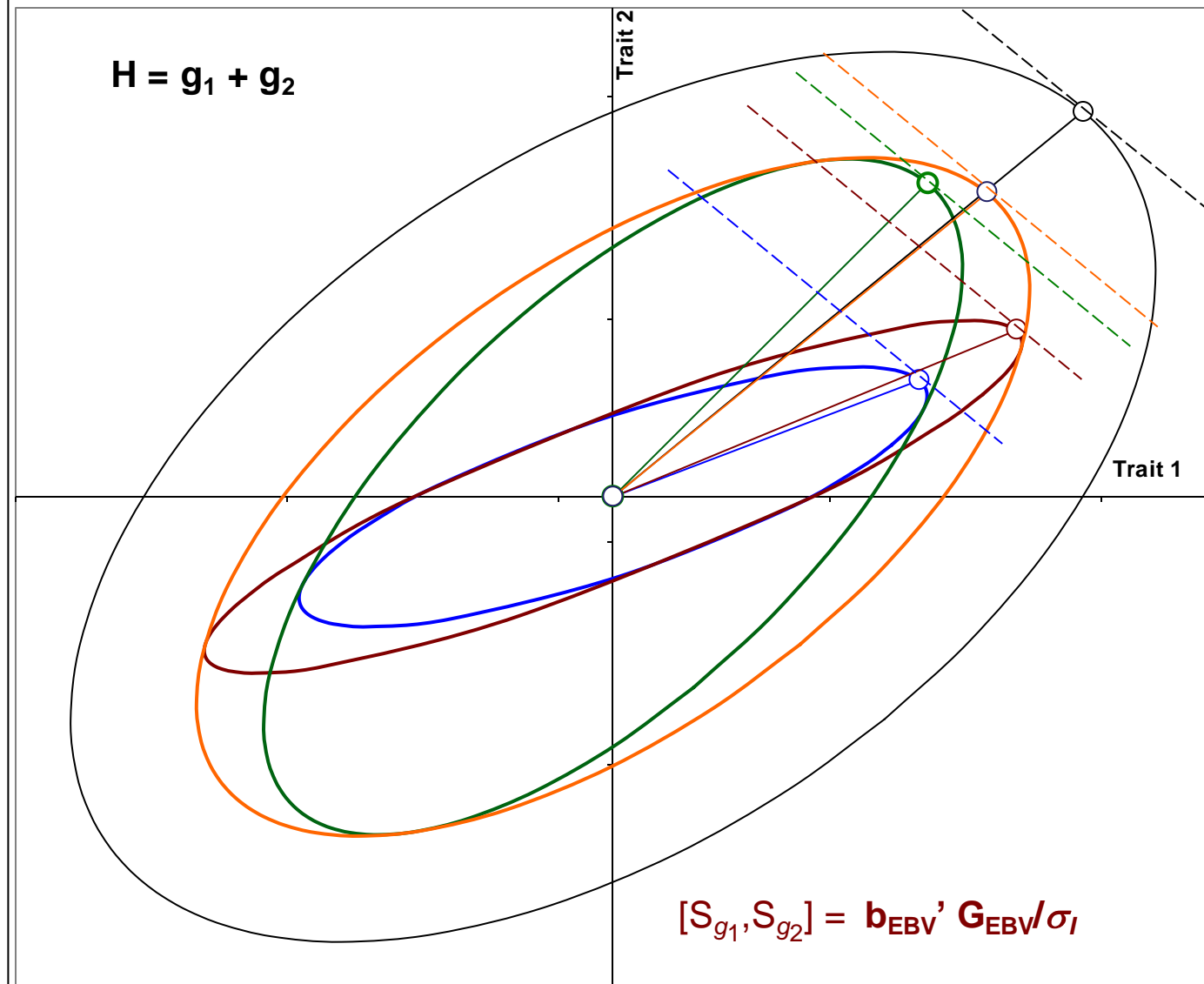
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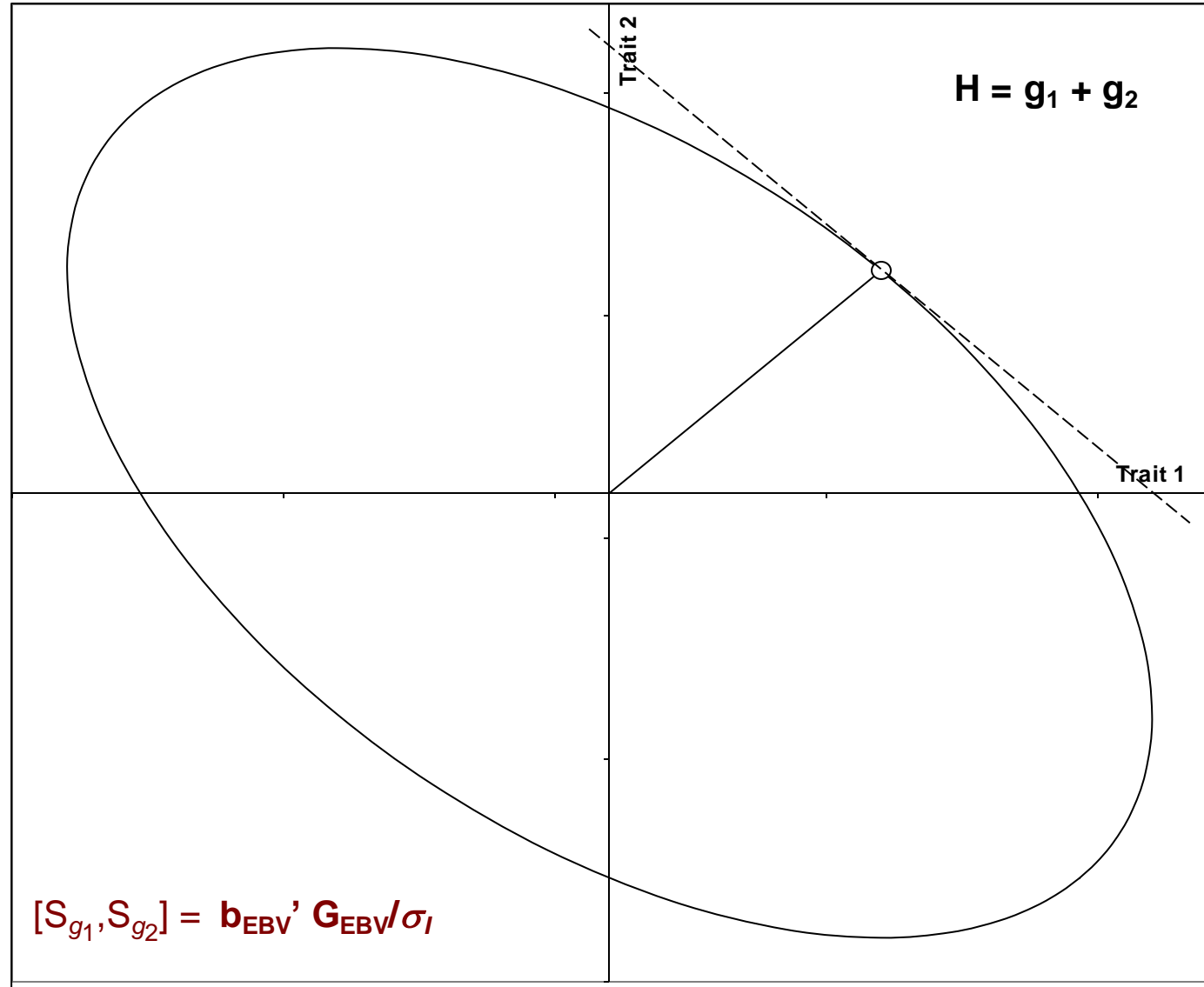
Use of Genomics to Shape Artificial Evolution

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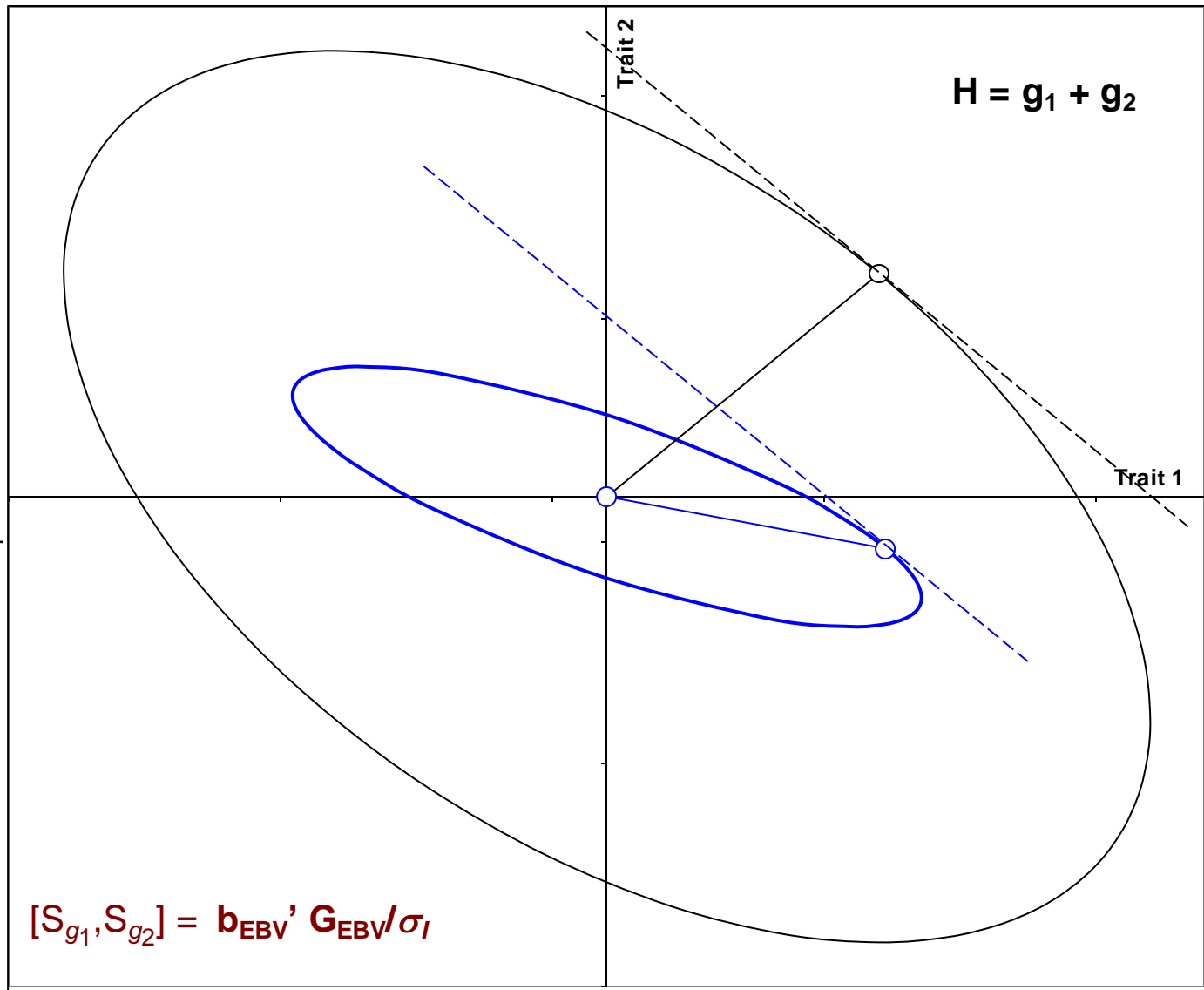
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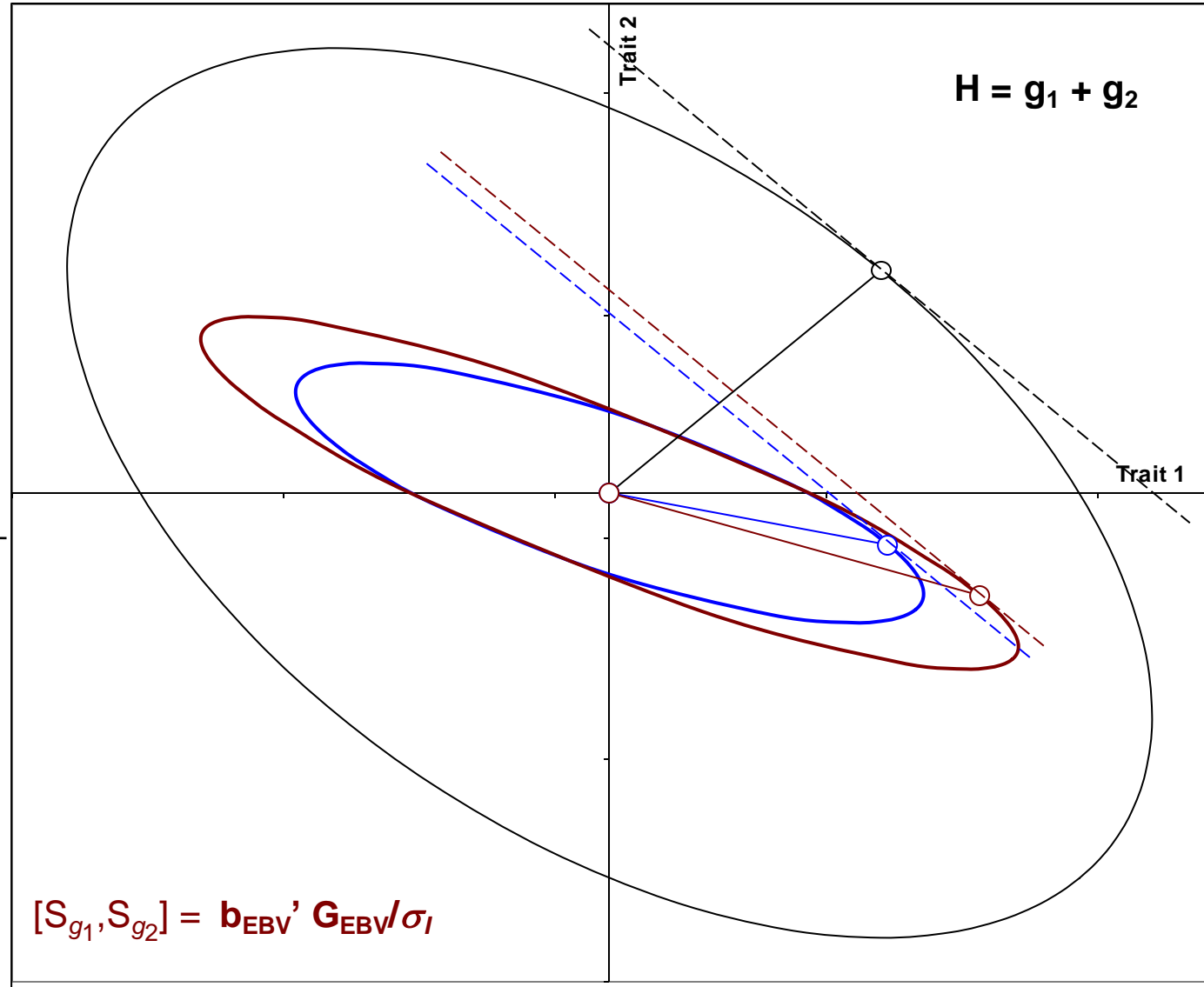
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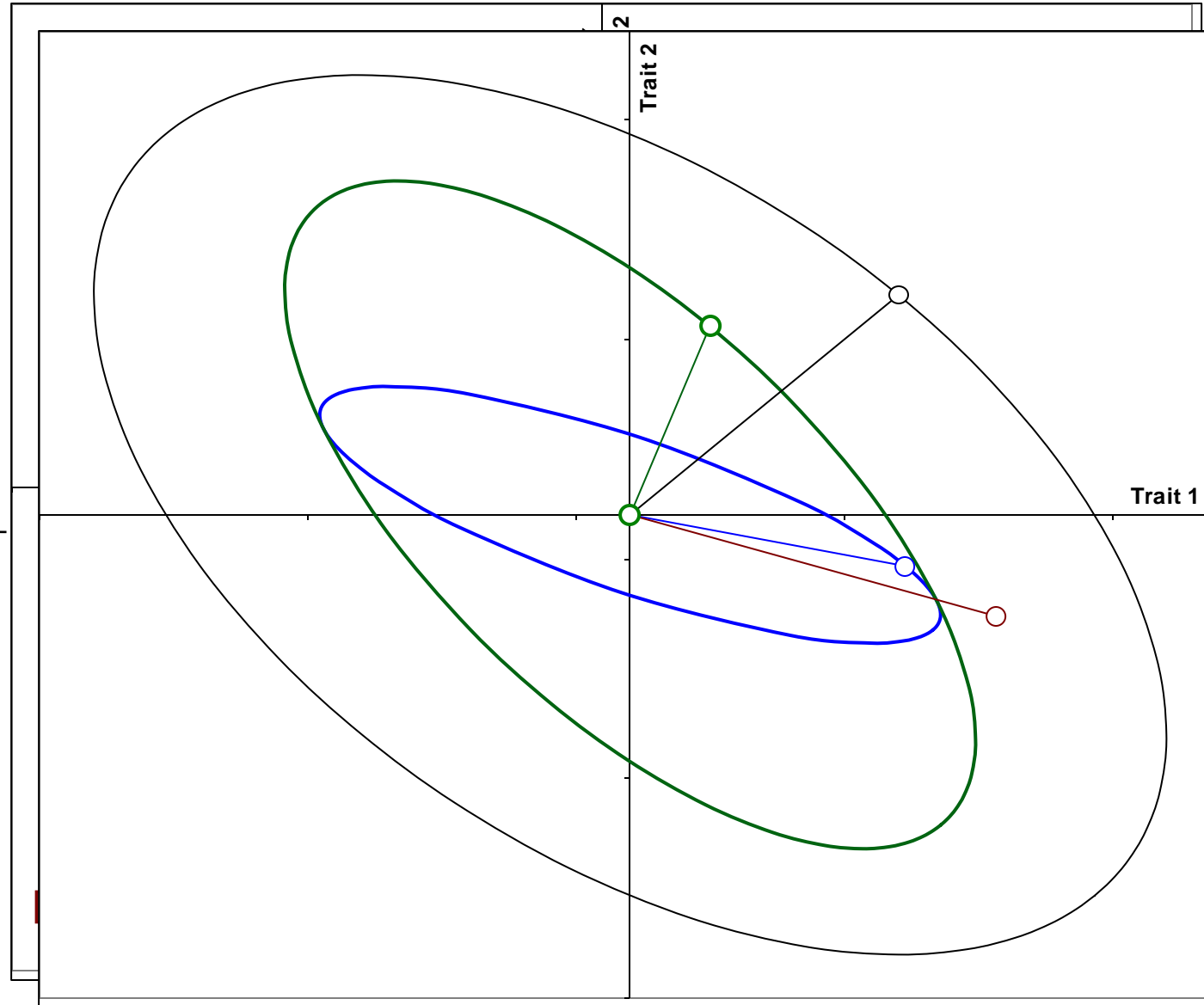
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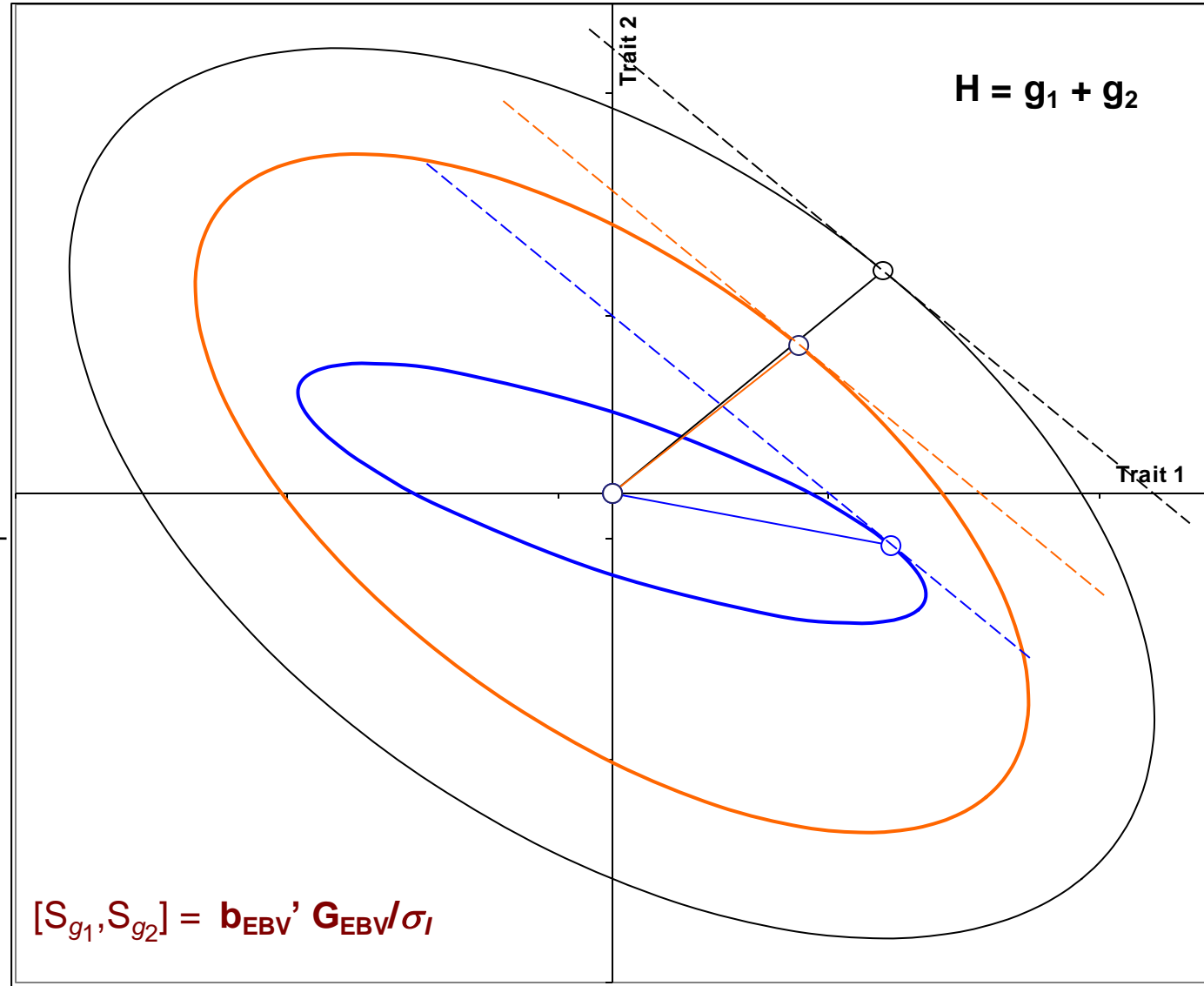
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	TS	TS
		
		
	GS	GS



Impact of Genomics on Response to Selection

Accuracy of GEBV for trait 2 to achieve same response in H as having a GEBV for trait 1 with accuracy 0.75.

Availability and accuracy of GEBV		$r_g = r_p$	Responses to selection			% increase in H
trait 1	trait 2		Trait 1	Trait 2	H	
-	-	0.5	0.56	0.26	0.83	35.1
0.75	-		0.74	0.37	1.12	
-	0.75		0.58	0.71	1.28	
0.75	0.75		0.69	0.68	1.37	
-	0.60		0.56	0.55	1.12	35.1
-	-	0	0.51	0.09	0.59	31.3
0.75	-		0.71	0.06	0.78	
-	0.75		0.32	0.61	0.93	
0.75	0.75		0.52	0.54	1.06	
	0.55		0.39	0.39	0.78	31.3
-	-	-0.5	0.51	-0.12	0.40	13.6
0.75	-		0.68	-0.23	0.45	
-	0.75		0.15	0.42	0.58	
0.75	0.75		0.35	0.33	0.68	
-	0.43		0.38	0.07	0.45	13.6

Using ST EBVs in a multi trait index

The index weights are $\mathbf{b} = \text{var}(\hat{\mathbf{a}})^{-1} \text{cov}(\hat{\mathbf{a}}, \mathbf{a}) \cdot \mathbf{v}$.

where \mathbf{a} are the true breeding values of the traits.

$\text{var}(\hat{\mathbf{a}})$ is an n by n matrix with

diagonals = $r_i \cdot \sigma_{a_i}^2$ where $\sigma_{a_i}^2 = G(i, i)$

and

off-diagonals = $r_i \cdot r_j \cdot \sigma_{a_{ij}} + 4r_i r_j \cdot (\sigma_{p_{ij}} - \frac{1}{4}\sigma_{a_{ij}}) / n$

Why are index weights different from economic weights?

Example: Protein Yield \$6.00/kg; Feed Intake -\$4.00/kg
Feed Intake not measured and not in Breed Obj:

$$\text{economic value PY} = \$6.00 + \boxed{0.5} \cdot (-\$4.00) = \$4.00$$

Genetic regression

Note: MTEBV, Feed Intake $0.5 \cdot \text{ST_EBV}_{\text{Protein}}$

multiple trait BLUP EBVs

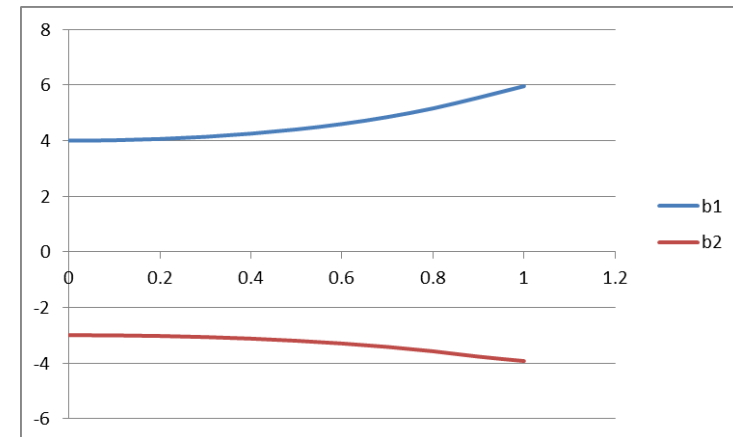
$$\begin{aligned} \text{Mt BLUP Index} &= 6 \cdot \text{MT_EBV}_{\text{Protein}} - 4 \cdot \text{MT_EBV}_{\text{FeedIntake}} \\ &= 4 \cdot \text{MT_EBV}_{\text{Protein}} \end{aligned}$$

But not Index = $6 \cdot \text{ST_EBV}_{\text{Protein}}$

Index weights for single trait EBVs depending on EBV accuracies

economic values 6 and -4 and a genetic regression of 0.5

EBVAccT1	EBVAccT2	IndWGhtT1	IndWghtT2
0.90	0.00	4.00	NA
0.90	0.50	4.40	-2.93
0.90	0.70	4.83	-3.12
0.90	0.90	5.50	-3.42
0.95	0.95	5.73	-3.69
0.99	0.99	5.96	-3.95
0.999	0.999	5.99	-3.99



Explaining the Index weights for ST_EBVs

In an index of ST_EBVs an index weight for a trait (say trait 1):

$$b_1 \text{ EV}_1 + b_2 \text{ EV}_2$$

where:

$$b_1 \sim r_{\text{ST_EBV}_1}^2 / r_{\text{MT_EBV}_1}^2$$

$$b_2 \sim (1 - r_{\text{ST_EBV}_2}^2) * \text{cov}(A_1, A_2) / \text{var}(A_1)$$

genetic regression

Index weights adjust for correlated response not accounted for by the non-perfect accuracy of a correlated trait

Effect of using incorrect (econ) weights on Selection Response

Case	Weight Used	Wght T1	Wght T2	\$Resp T1	\$Resp T2	Tot \$Resp	Rel Resp
1	EV	1.00	2.00	0.36	0.61	0.96	96.5%
	IW	1.77	1.76	0.42	0.58	1.00	
2	EV	1.00	1.00	0.41	0.29	0.71	99.2%
	IW	1.38	0.92	0.43	0.28	0.71	

STEBV accuracies 0.9 and 0.5 and a genetic correlation of 0.5.

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2	EV	1.00	1.00	0.41	0.29	0.71	99.2%
	IW	1.38	0.92	0.43	0.28	0.71	
3	EV	1.00	-1.00	0.37	-0.08	0.29	99.2%
	IW	0.60	-0.75	0.34	-0.05	0.29	

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	IW	0.60	-0.75	0.34	-0.05	0.29	
4	EV	1.00	-1.25	0.34	-0.06	0.28	96.5%
	IW	0.50	-0.96	0.25	0.04	0.29	

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	IW	0.50	-0.96	0.25	0.04	0.29	
5	EV	1.00	-1.50	0.30	-0.02	0.28	91.9%
	IW	0.40	-1.17	0.15	0.16	0.31	

STEBV accuracies 0.9 and 0.5 and a genetic correlation of 0.5.

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	IW	0.60	-0.75	0.34	-0.05	0.29	
4	EV	1.00	-1.25	0.34	-0.06	0.28	96.5%
	IW	0.50	-0.96	0.25	0.04	0.29	
5	EV	1.00	-1.50	0.30	-0.02	0.28	91.9%
	IW	0.40	-1.17	0.15	0.16	0.31	
6	EV	1.00	-2.00	0.24	0.08	0.32	82.5%
	IW	0.21	-1.59	-0.02	0.41	0.39	

STEBV accuracies 0.9 and 0.5 and a genetic correlation of 0.5.

Using ST EBVs Summary

- When using ST EBVs in an index:
- index weights are economic weights but adjusted for correlated changes in other traits as far as those are not accounted for by their EBVs
- Additional weight is given to traits that are well measured,
- Includes value of response other breeding objective traits not well measured
- not making this adjustment ignores the value of some correlated changes
- Although the overall response is likely to be not affected very much (less than 5%), the weight and response for Production would be higher and the weight and response for Fertility or Fitness would be considerably lower compared to an optimal index

Summary ST EBVs in index

- When using ST EBVs, best approach is to use index weights, not straight economic weights
- But Index Weights can cause some practical problems
 - Some weights maybe hard to explain
 - Different bulls might need different weights
 - Do not publish weights? Then not easy to calculate your own index