Breeding Objectives

Multi-trait selection – how to implement?

- Need to combine
  - the relative economic weights
  - genetic parameters (heritabilities, correlations)

to determine the weights we put on the observed phenotypes

$$\text{Index} = b_1X_1 + b_2X_2$$

Issues with MT selection

- We have to spread our selection efforts over several traits, each of them weighted economically

- Selection for one trait gives also a correlated response for other traits

We need weights for selection criteria

- Index = $b_1X_1 + b_2X_2 + \ldots + b_nX_n$

Selection index with more information sources (multiple regression)

X = vector with phenotypes (criteria)

A = aggregate genotype (single trait here)

$$\mathrm{var}(X) = P = \text{matrix} = \begin{bmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1, X_2) \\ \mathrm{cov}(X_2, X_1) & \mathrm{var}(X_2) \end{bmatrix}$$

$$\mathrm{cov}(X, A) = G = \text{vector} = \begin{bmatrix} \mathrm{cov}(x_1, A) \\ \mathrm{cov}(x_2, A) \end{bmatrix}$$

weights: $b = P^{-1}G$

Selection index with more information sources and with more objective traits (multiple regression)

X = vector with phenotypes (criteria)

H = aggregate genotype (multiple traits here)

$= v_1 A_1 + v_2 A_2$

$$\mathrm{var}(X) = P = \text{matrix} = \begin{bmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1, X_2) \\ \mathrm{cov}(X_2, X_1) & \mathrm{var}(X_2) \end{bmatrix}$$

$$\mathrm{cov}(X, A) = G = \text{matrix} = \begin{bmatrix} \mathrm{cov}(X_1, A_1) & \mathrm{cov}(X_1, A_2) \\ \mathrm{cov}(X_2, A_1) & \mathrm{cov}(X_2, A_2) \end{bmatrix}$$

weights: $b = P^{-1} G v$

Index weights example

	σ_p	h^2	rg	rp	a	b
FW	.4	.4			5	2
			0	0		
FD	2	.4			-1	-0.4

Heritabilities same and no correlation;

Weights are proportional to rel. economic weight

Index weights example

	σ_p	h^2	rg	rp	a	b
FW	.4	.3			5	1.5
			0	0		
FD	2	.5			-1	-0.5

More weight for traits with higher heritability

Index weights example

	σ_p	h^2	rg	rp	a	b
FW	.4	.3			5	0.53
			0.5	0		
FD	2	.5			-1	-0.31

Weights also depend on correlations

In general, weights on phenotypic information sources are not easy to 'recognize'

Selection index for Single Trait

$$\mathrm{var}(X) = P = \text{matrix} = \begin{bmatrix} \mathrm{var}(x_1) & \mathrm{cov}(x_1, x_2) \\ \mathrm{cov}(x_2, x_1) & \mathrm{var}(x_2) \end{bmatrix}$$

$$\mathrm{cov}(X, A) = G = \text{vector} = \begin{bmatrix} \mathrm{cov}(x_1, A) \\ \mathrm{cov}(x_2, A) \end{bmatrix}$$

weights: $b = P^{-1}G$ gives weight for all sources about one EBV

Selection index for multiple traits

H = aggregate genotype = $v_1 A_1 + v_2 A_2$

var(X) = P = matrix = $\begin{bmatrix} \mathrm{var}(x_1) & \mathrm{cov}(x_1, x_2) \\ \mathrm{cov}(x_2, x_1) & \mathrm{var}(x_2) \end{bmatrix}$

cov$(X$,A) = G = matrix = $\begin{bmatrix} \mathrm{cov}(x_1, A_1) & \mathrm{cov}(x_1, A_2) \\ \mathrm{cov}(x_2, A_1) & \mathrm{cov}(x_2, A_2) \end{bmatrix}$

weights: b = $P^{-1}Gv$ = $[b_1 \quad b_2] \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

b_i is a subset of weights for i^{th} trait to give EBV_i

Overall weights are weighting each subset with its economic weight

Using EBV's rather than own phenotypes as selection criteria

$$\text{Index} = v_1 EBV_1 + v_2 EBV_2 + \ldots + v_n EBV_n$$

weights are equal to economic values!

as genetic parameters are already accounted for in MT-BLUP generation of EBV's

Index selection is more efficient than single trait selection!

Predicting response to MT selection

- Response in dollars:

$$\mathbf{R = i.\sigma_{Index} = i.\sqrt{b'Pb}}$$

Response for each trait

$$\mathbf{[R_1\ R_2\ \ldots R_m] = i.b'G/\sqrt{b'Pb}}$$

Are selection indices always linear?

- nonlinear profit function
- optimal traits
- threshold values for profit

Selection index with ‘desired gains’

- Rather than
  - determine econ. values >>>> response
  - We desire a response >>>> economic values (implicit)

When useful?

Predicting genetic change to multiple trait selection

- Single trait selection response
- Correlated response to selection
- Response to index selection
  - How can multiple trait response be manipulated by varying index weights
  - Can we go anywhere we want?

Direct response to single trait selection

$R = i.h^2.\sigma_P$ if mass selection

$R = i.r_{IA}.\ \sigma_A$ more general:

r_{IA} is accuracy of selection

Direct and Correlated response to single trait selection

Response = $i.h_1.\sigma_{A1}$

and

Correlated Response = $i.h_1.r_g.\sigma_{A2}$

single trait selection!

Combining information on two traits

selection index **I = $b_1X_1 + b_2X_2$**

$$P = \mathrm{var}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1, X_2) \\ \mathrm{cov}(X_2, X_1) & \mathrm{var}(X_2) \end{pmatrix} = \begin{pmatrix} s_{p_1}^2 & r_p s_{p_1} s_{p_2} \\ r_p s_{p_1} s_{p_2} & s_{p_2}^2 \end{pmatrix}$$

$$G = \mathrm{cov}\left(\begin{matrix} X_1 \\ X_2 \end{matrix}, A_1\right) = \begin{pmatrix} \mathrm{cov}(X_1, A_1) \\ \mathrm{cov}(X_2, A_1) \end{pmatrix} = \begin{pmatrix} s_{A_1}^2 \\ r_g s_{A_1} s_{A_2} \end{pmatrix}$$

$$b = P^{-1}G = \begin{pmatrix} 11.5 & 21.9 \\ 21.9 & 145 \end{pmatrix}^{-1} \begin{pmatrix} 5.75 \\ 2.74 \end{pmatrix} = \begin{pmatrix} 0.6517 \\ 0.0796 \end{pmatrix}$$

Multiple Trait breeding goal

- Aggregate genotype: $H = v_1 A_1 + v_2 A_2$
- selection index $I = b_1 X_I + b_2 X_2$

Multiple Trait breeding goal

- **Aggregate genotype:** $\mathbf{H = v_1 g_1 + v_2 g_2}$
- **selection index** $\mathbf{I = b_l X_l + b_2 X_2}$

$$G = \operatorname{cov}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, (A_1 \quad A_2) = \begin{pmatrix} \operatorname{cov}(X_1, A_1) & \operatorname{cov}(X_1, A_2) \\ \operatorname{cov}(X_2, A_1) & \operatorname{cov}(X_2, A_2) \end{pmatrix}$$

$$= \begin{pmatrix} s^2_{g_1} & r_g s_{A_1} s_{A_2} \\ r_g s_{A_1} s_{A_2} & s^2_{A2} \end{pmatrix} = \begin{pmatrix} 5.75 & 2.74 \\ 2.74 & 14.5 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = P^{-1} G a = \begin{pmatrix} 11.5 & 22.05 \\ 22.05 & 145 \end{pmatrix}^{-1} \begin{pmatrix} 5.75 & 2.74 \\ 2.74 & 14.5 \end{pmatrix} \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} = \begin{pmatrix} 0.618 \\ -0.125 \end{pmatrix}$$

- **Variance of index:**

  $s_I^2 = b'Pb = 3.27$ **$$**

- **Variance of the aggregate genotype**

  $s_H^2 = v'Cv = 6.64$ **$$** **C = var(A) ..of breeding values**

- **Accuracy of Index:**

  $r_{IH} = s_I / s_H =$ Ö(b'Pb /v'Cv)

- **Response to selection**

  $R = i.r_{IH}.\sigma_A = i.\sigma_I$ in $$

- **Response in each trait:**

  $\delta g_i = b_{gi,I}\, R$

  $= i.b'G_i / \sigma_I$

- **Notice that sum of** $\delta g_i . v_i = R$

See also mtindex.xls

Example

body weight $h^2_A = 0.40$ $s_P = 17$ kg

feed intake $h^2_B = 0.25$ $s_P = 2.0$ kg

$r_g = .50$ $r_p = 0.20$

selection intensity=1.0

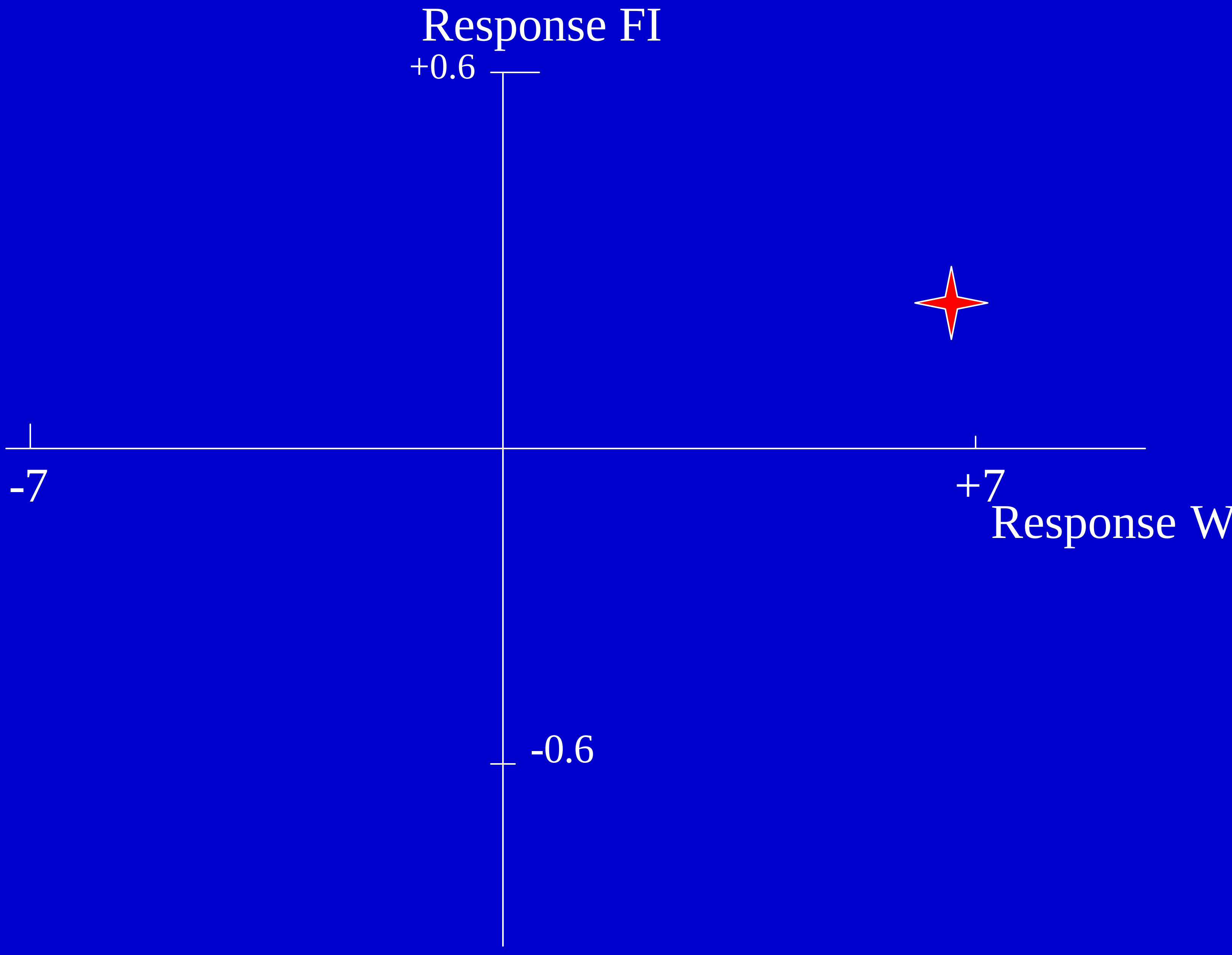
Criteria for selection | True Breeding Value

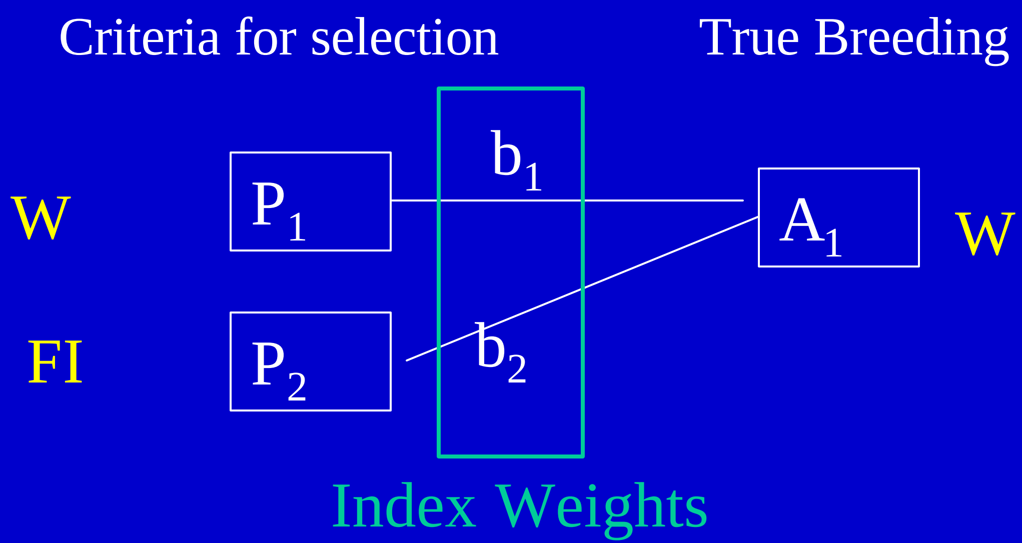
W $\boxed{P_1} \xrightarrow{b_1} \boxed{A_1}$ W

Index = EBV = $0.4P_W$

Response = **6.80** kg Weight

Correl. Resp. = **0.32** kg Feed Intake

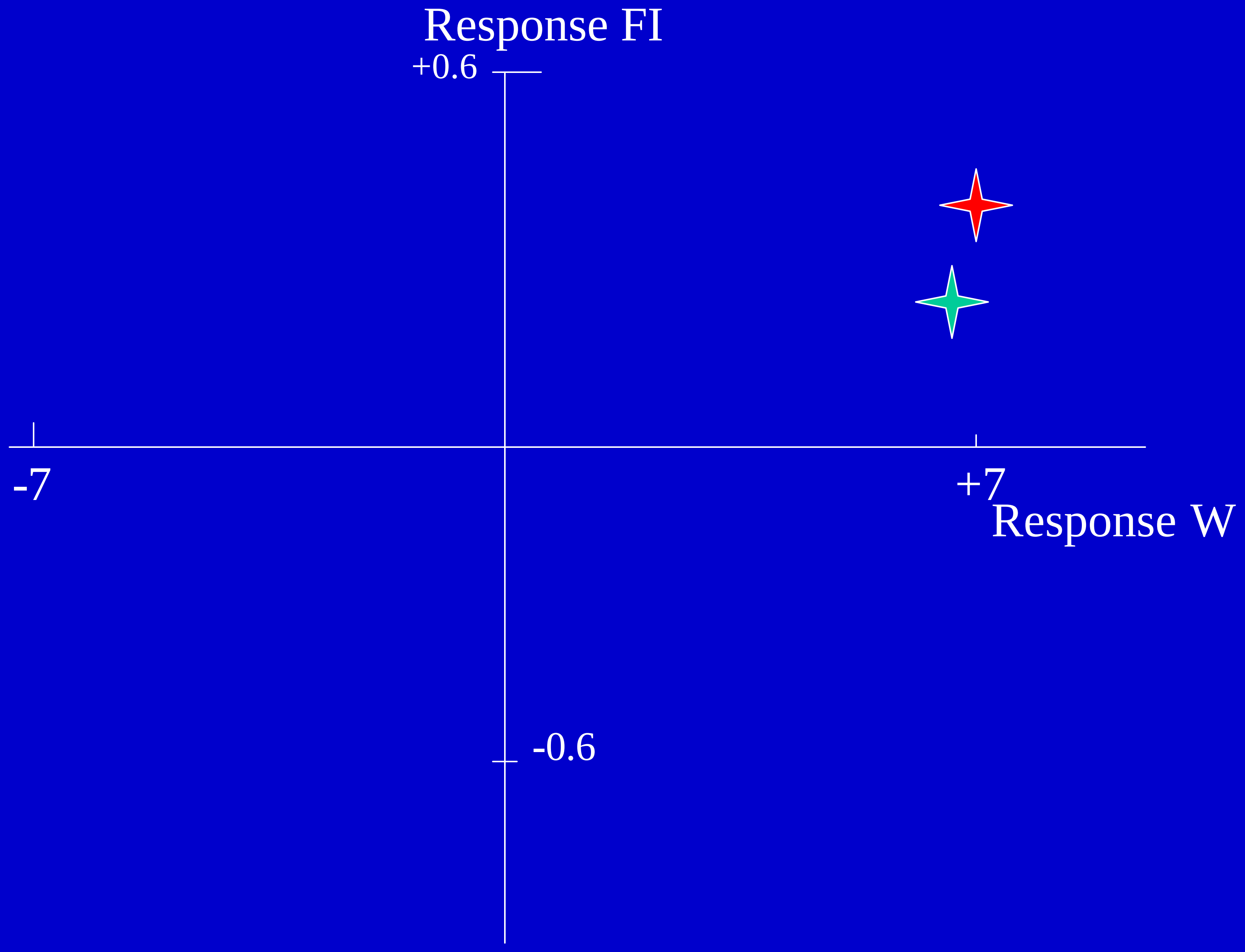

Response FI
+0.6
-7
+7
Response W
-0.6



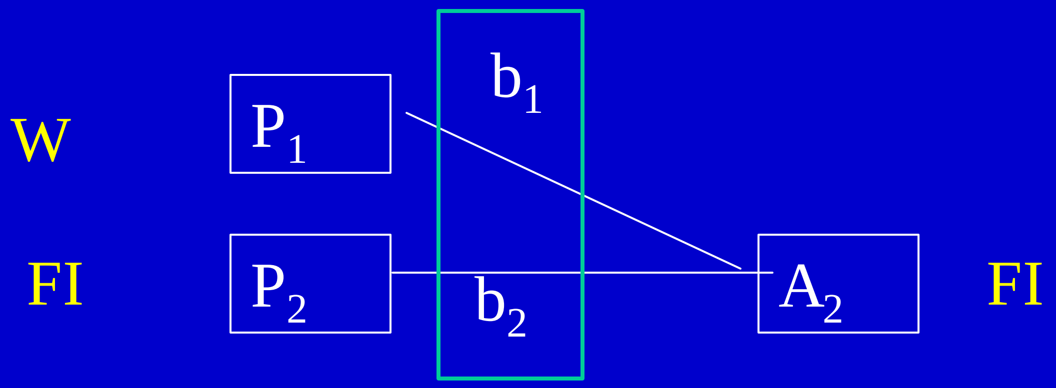
Index = \$EBV = $0.38P_{W} + 0.69P_{FI}$

R_{W} = **6.93** kg

R_{FI} = **0.40** kg


Response FI
+0.6
-7
+7
Response W
-0.6

Criteria for selection True Breeding Value

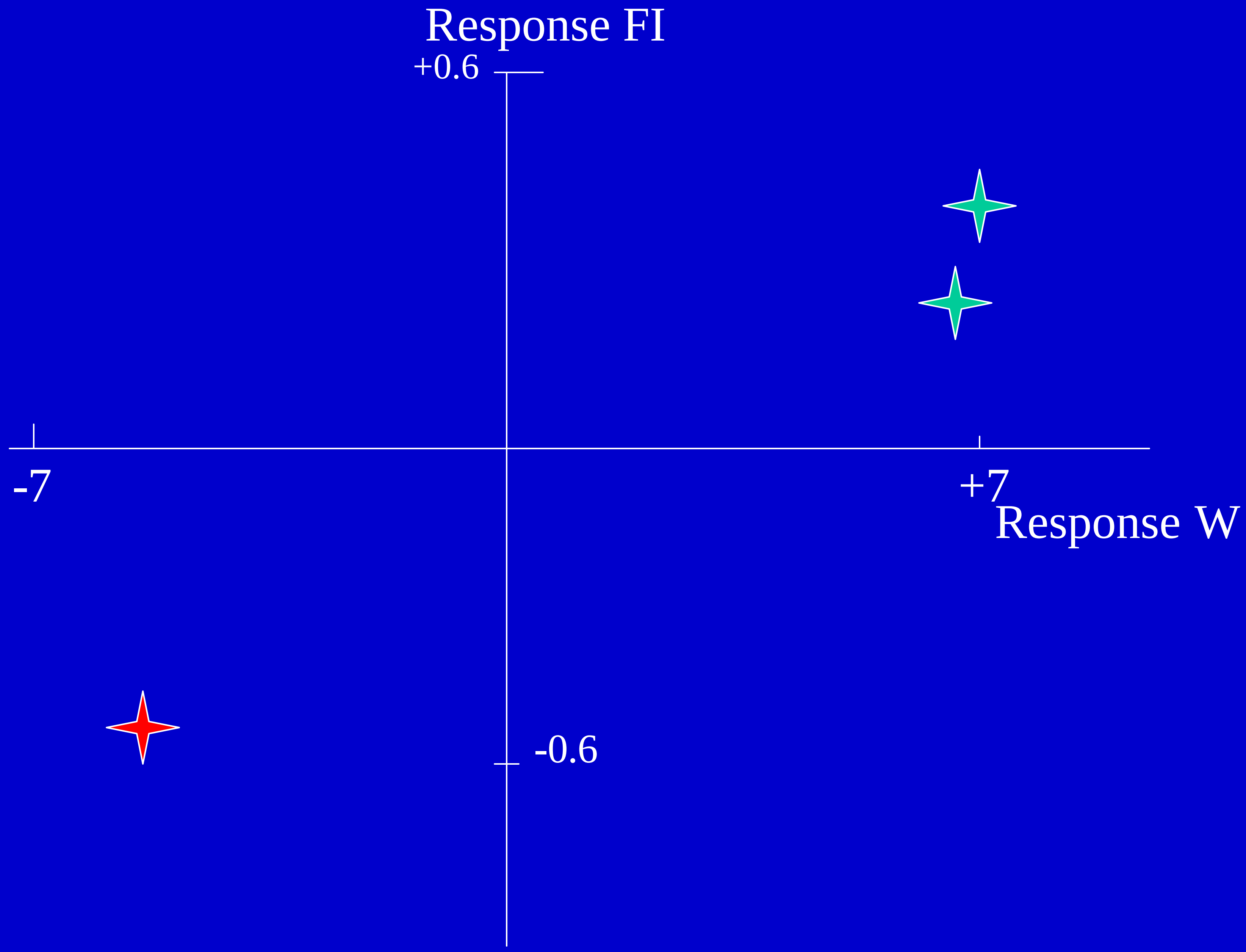


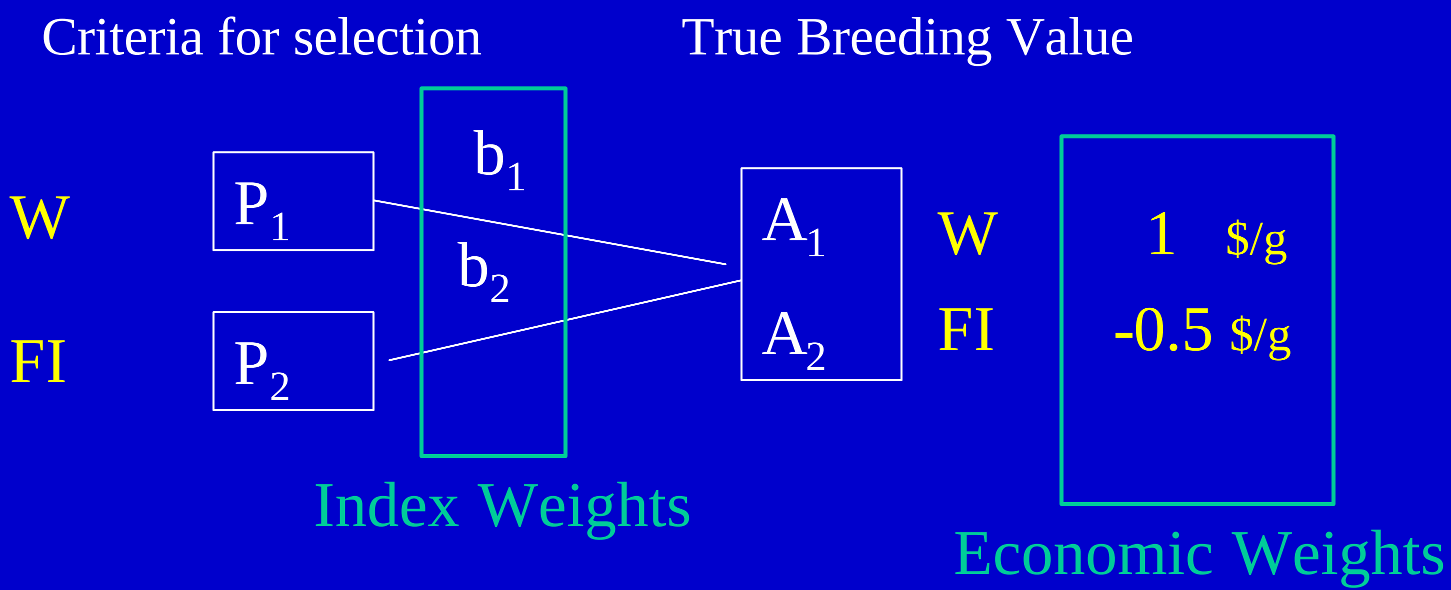
Index Weights

$$\text{Index} = \$\text{EBV} = -0.013P_{\mathrm{W}} - 0.23P_{FI}$$

R_{W} = **-5.04** kg

R_{FI} = **-0.55** kg

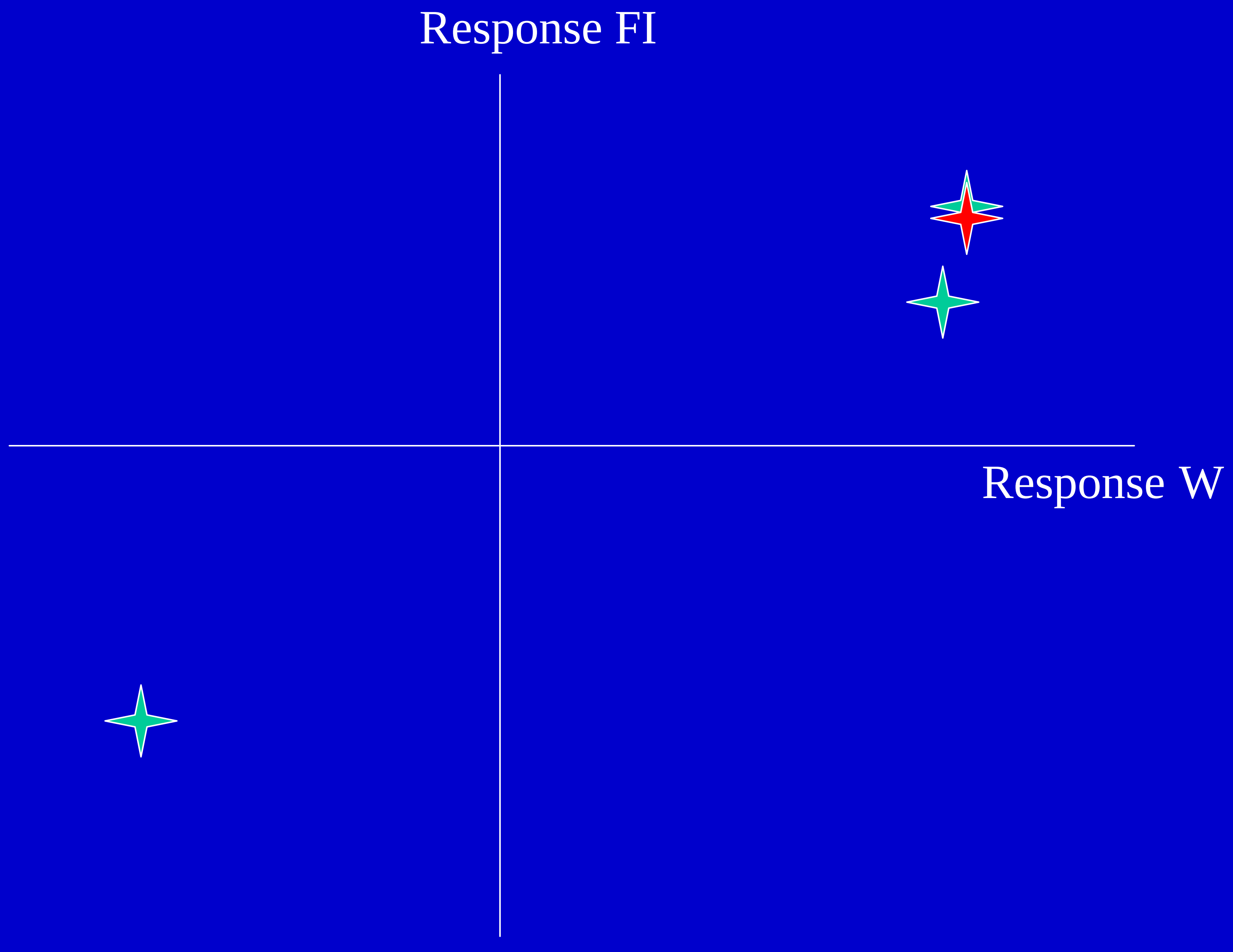

Response FI
+0.6
-7
+7
Response W
-0.6

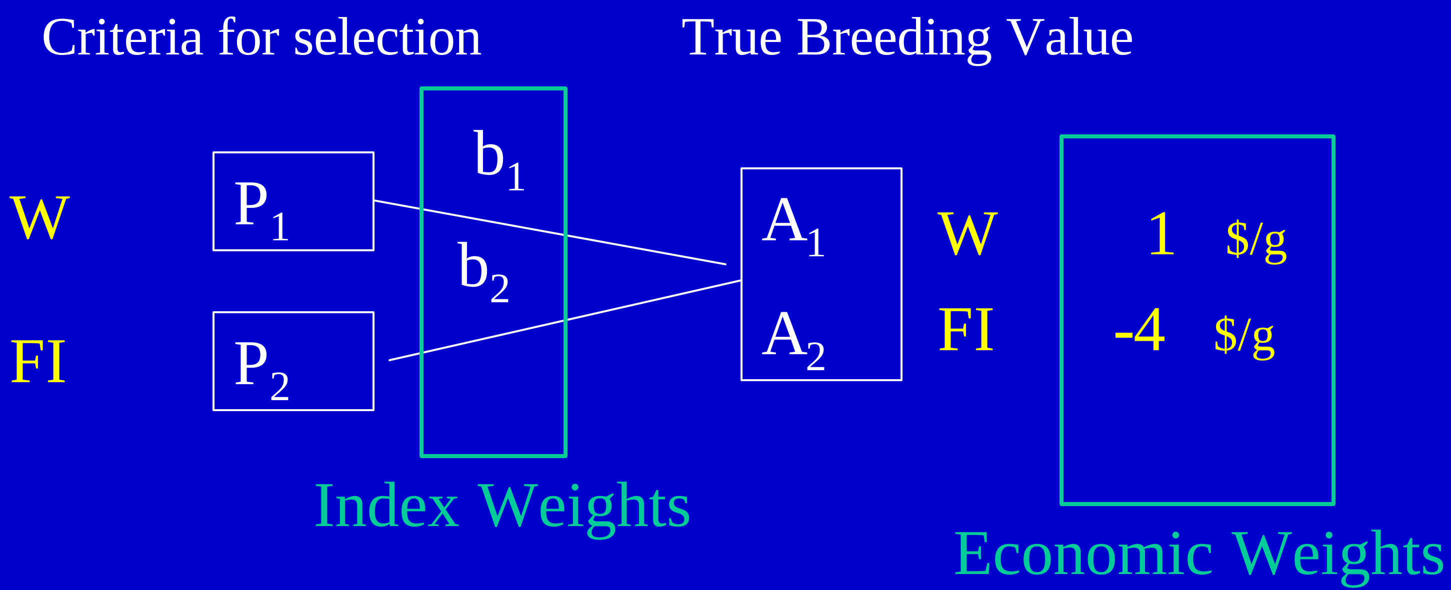


$$\text{Index} = \$\text{EBV} = 0.62P_W - 0.13P_{FI}$$

R_W = **6.93** kg

R_{FI} = **0.39** kg

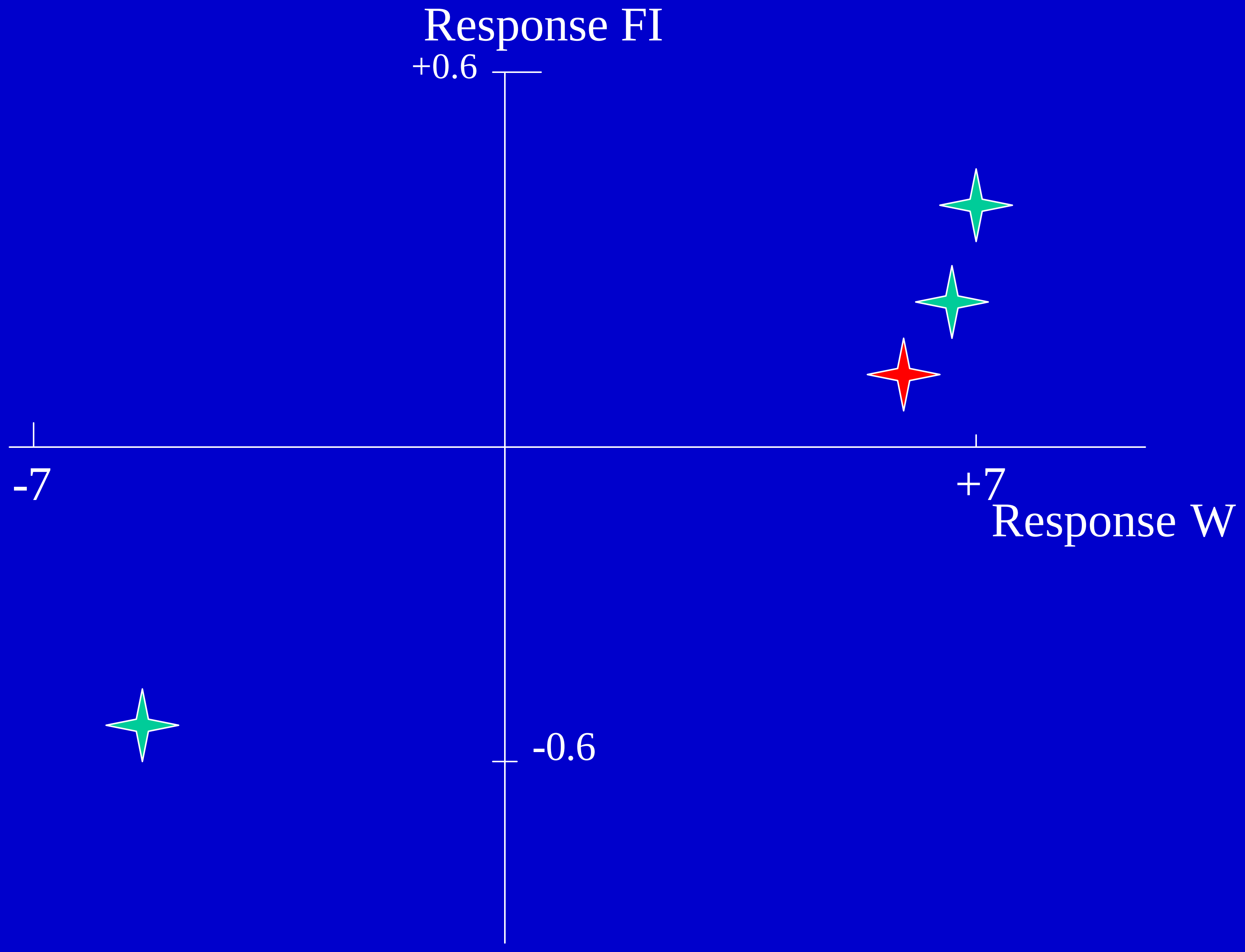

Response FI
Response W



Index = \$EBV = $0.33P_W - 0.22P_{FI}$

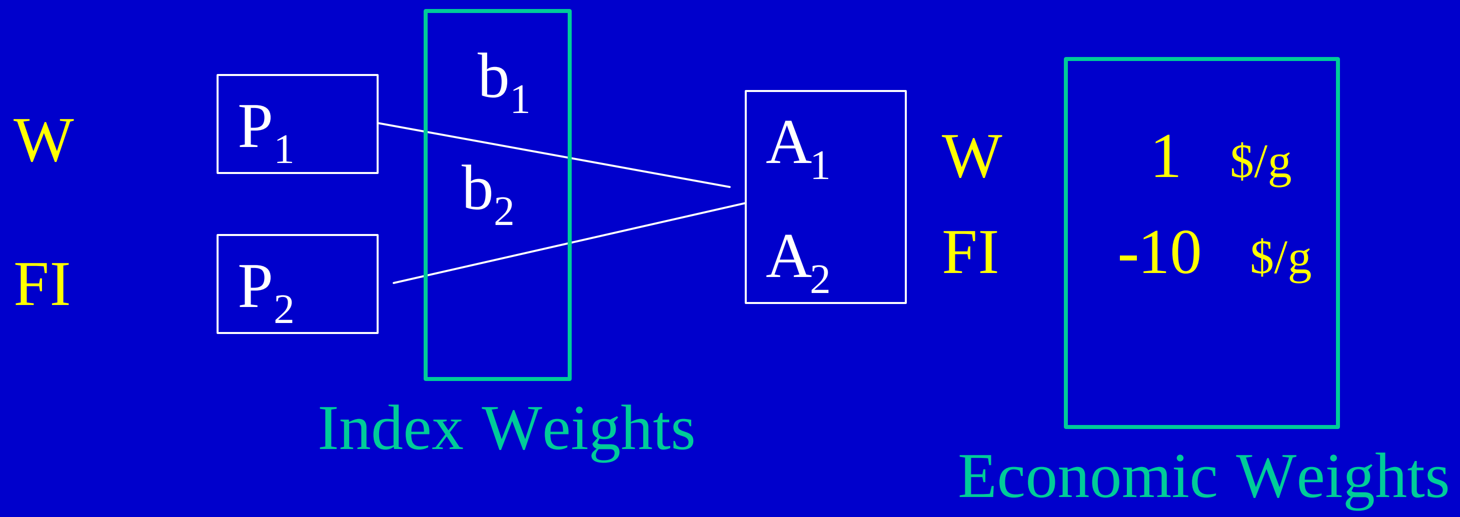
R_W = **6.68** kg

R_{FI} = **0.28 k**g



Response FI
+0.6
-7
+7
Response W
-0.6

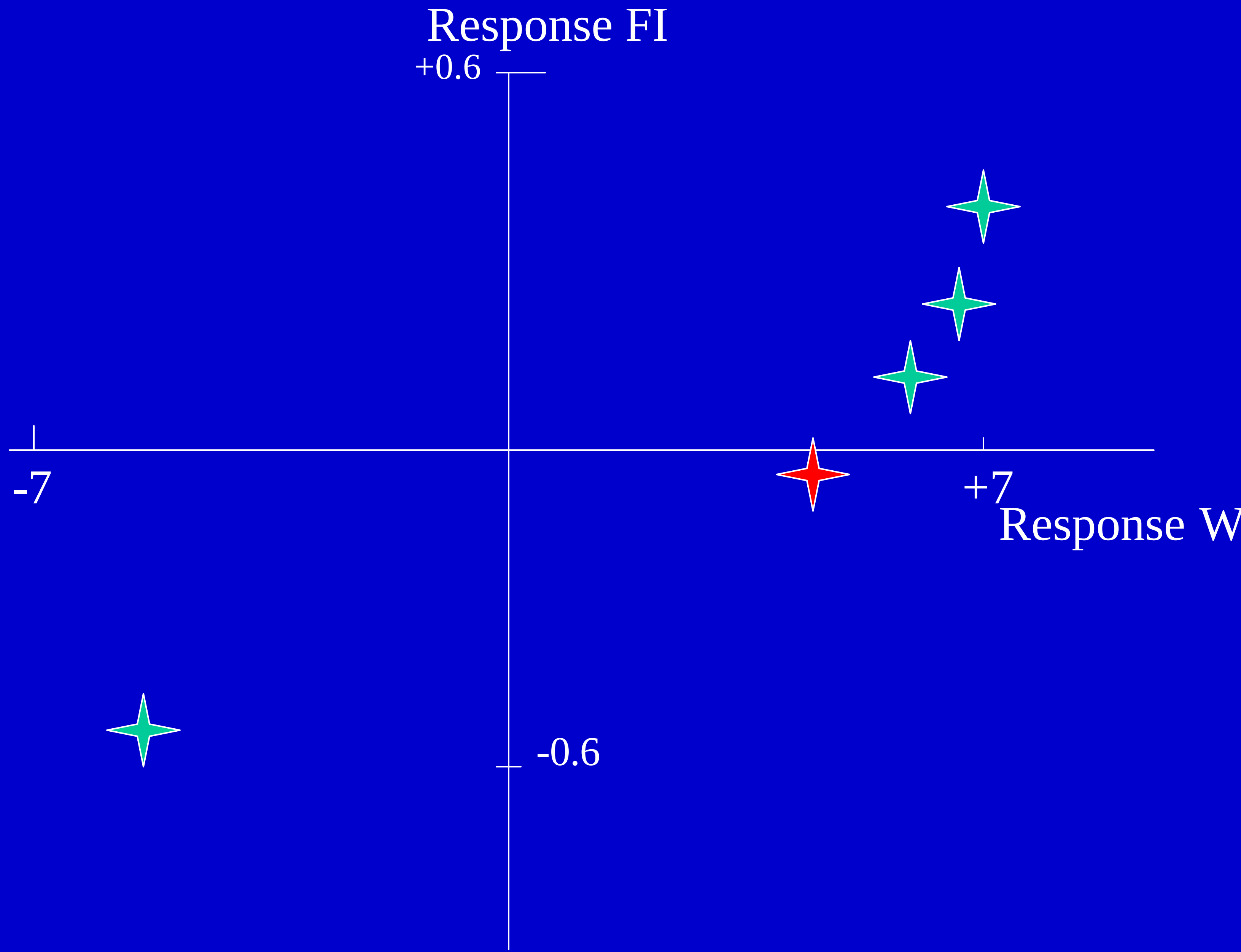
Criteria for selection True Breeding Value



Index = \$EBV = $0.25P_W - 1.58P_{FI}$

R_W = **4.29** kg

R_{FI} = **-0.05 k**g

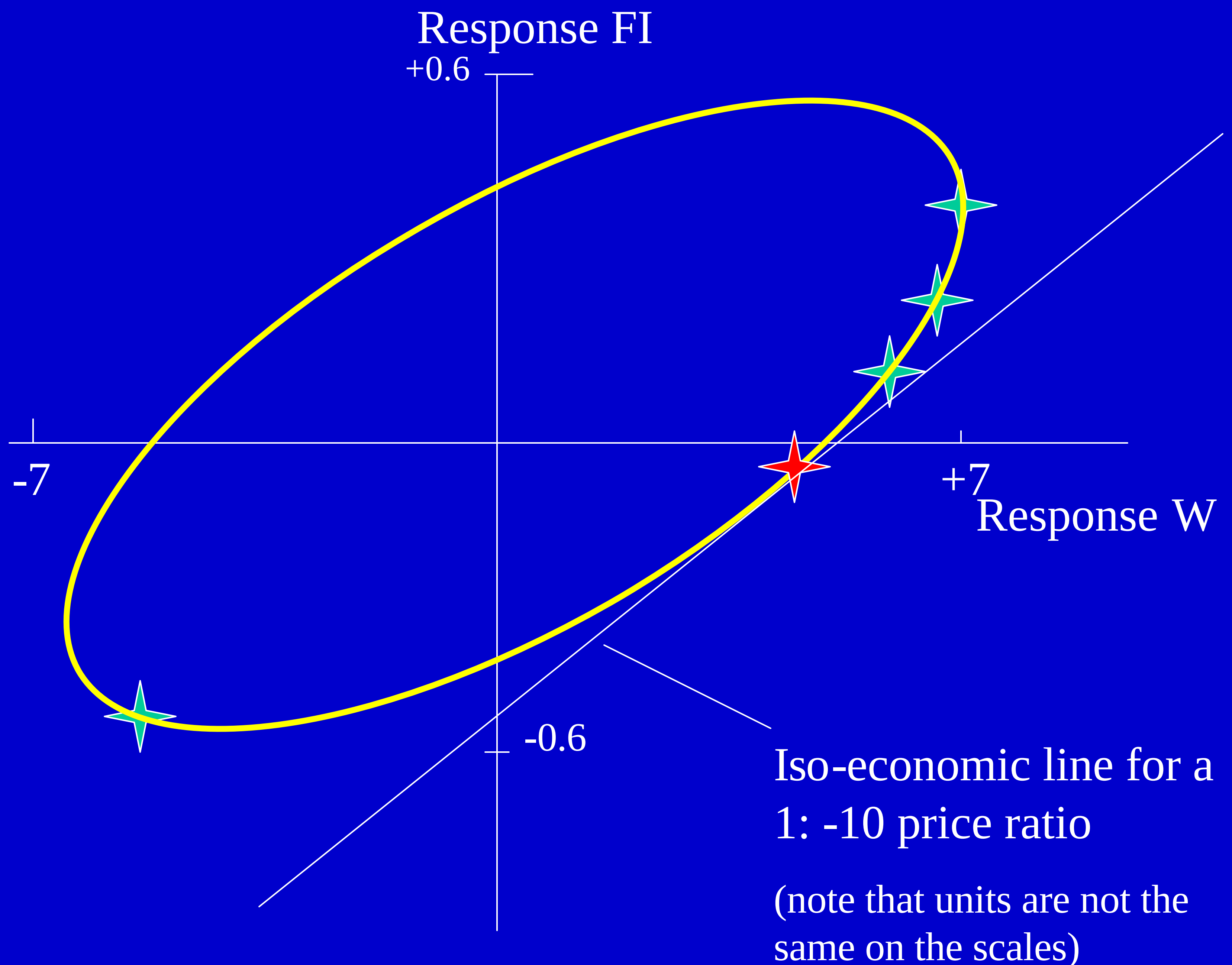

Response FI
+0.6
-7
+7
Response W
-0.6

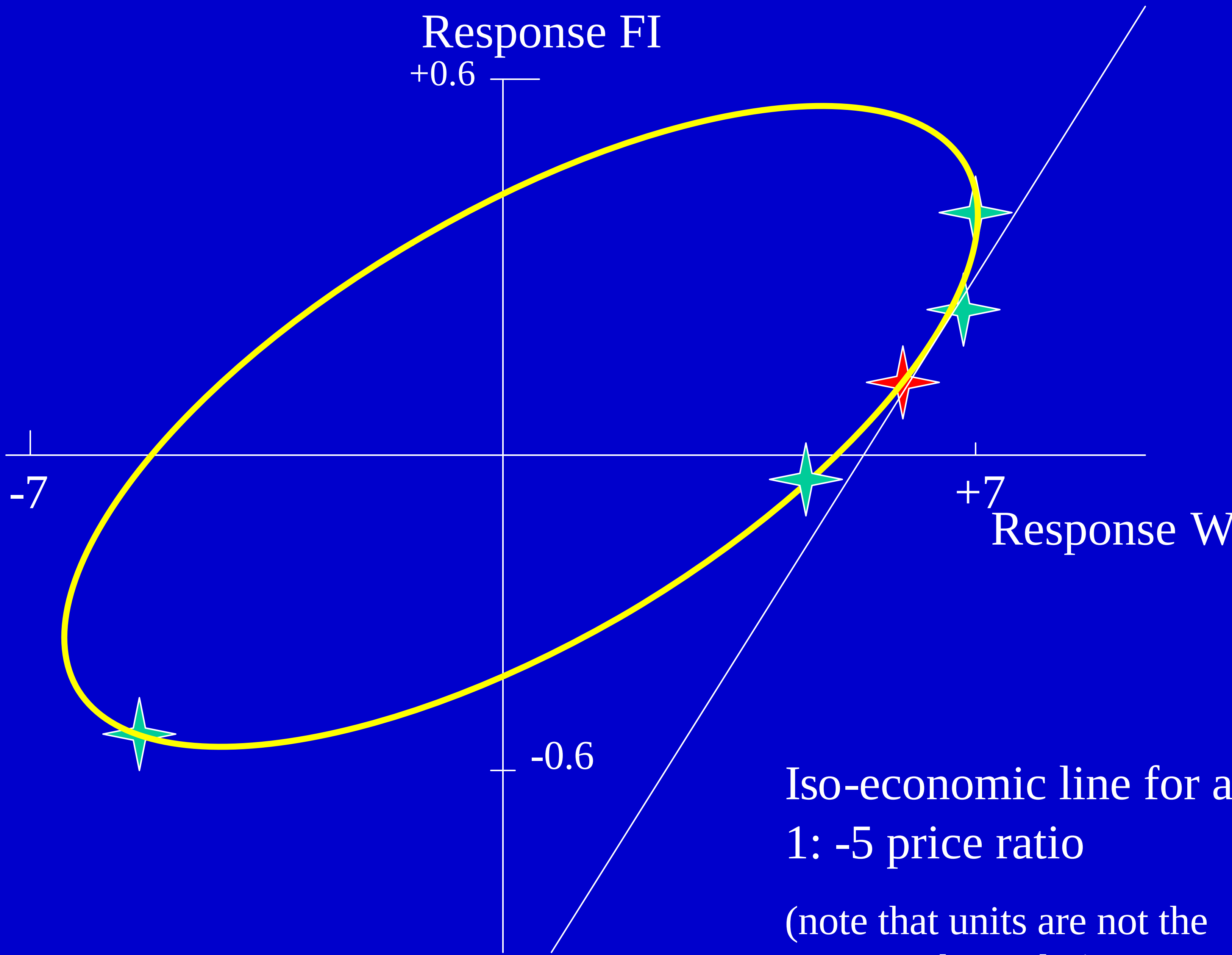
Summary of some possible responses

Information on	breeding goal		R. weight	R. feed intake
	a_1	a_2		
Weight	1	0	6.80	0.32
Feed	0	-1	-2.69	-0.50
Weight + feed	1	0	6.93	0.40
Weight + feed	0	-1	-5.64	-0.55
Weight + feed	0	-0.5	-5.93	0.39
Weight + feed	1	-1	6.92	0.38
Weight + feed	1	-4	6.68	0.28
Weight + feed	1	-10	4.29	-0.05
Weight + feed	1	-20	-0.93	-0.43

Note: Optimal selection pre-determined economic values, response follows from that.

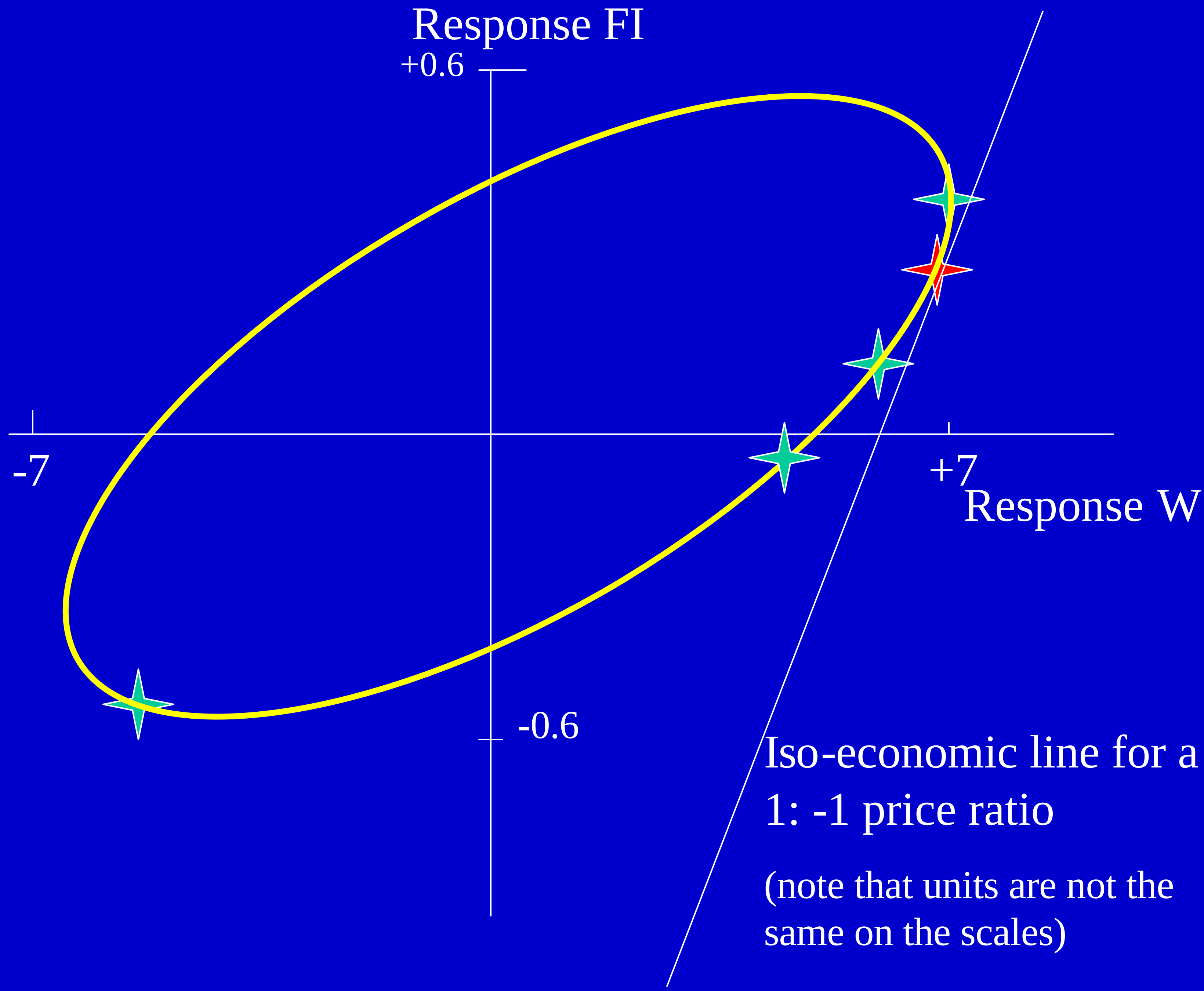
-otherwise: desired gains index
restricted index


Response FI
+0.6
-7
+7
Response W
-0.6
Iso-economic line for a
1: -10 price ratio
(note that units are not the
same on the scales)



Iso-economic line for a 1: -5 price ratio

(note that units are not the same on the scales)


Response FI
+0.6
-7
+7
Response W
-0.6
Iso-economic line for a
1: -1 price ratio
(note that units are not the
same on the scales)

Predicting genetic change to multiple trait selection

- Response to index selection
  - How can multiple trait response be manipulated by varying index weights
  - Can we go anywhere we want?