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ADVANCED METHODS IN GENOME ANALYSIS

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Day 1. Fine mapping and analysis of complex pedigrees

1. Combining linkage and linkage disequilibrium information
2. Analysis of crosses between outbred lines
3. QxPak software

Day 2. cDNA microarray analysis

1. Basic techniques

Clustering

2. Prediction of phenotype given cDNA pattern

Partial Least Squares

3. Genetical genomics

Heat shock proteins (rats)

Whole genome (yeast)

Combining expression and markers for gene detection

Genetics has become a data rich science, where the limiting step already *NOW* is the data analysis, rather than in the obtention of the data themselves.

Recall the first QTL experiments, the rationale behind was to measure as many traits as possible because we wanted to maximize the output per marker typed.

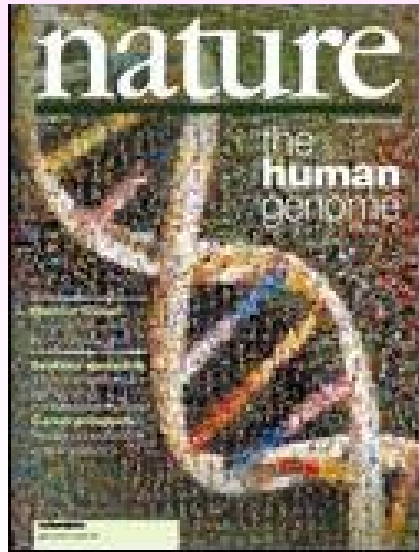
Three main streams of data:

DNA polymorphism (markers)

Expression data (functional genomics)

DNA Sequence

The culprit



What are quantitative traits?

Sensitive to the environment
Affected by several genes



Traits showing a continuous distribution

The classical framework ...

is composed of two distinct parts:

1. The mixed model.
2. The infinitesimal genetic model (Fisher, 1918).

The mixed model

The diagram shows the mixed model equation $y = Xb + Zu + e$ with callouts for each term:

- y**: phenotypes
- X**: incidence matrices
- b**: fixed effects
- Z**: incidence matrices
- u**: random effects
- e**: residuals

From

the mixed model theory

+

the infinitesimal genetic model

=

the mixed model equations (MME, Henderson, 1950)

$$\begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}'\mathbf{R}^{-1}\mathbf{Z} \\ \mathbf{Z}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}'\mathbf{R}^{-1}\mathbf{Z} + \lambda \mathbf{G}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}'\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}$$

The problem:

incidence matrix

gene effects

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \cancel{\mathbf{Z}\mathbf{u}} + \sum_{j=1}^{n_{\text{loci}}} \mathbf{W}_j \mathbf{g}_j + \mathbf{e}$$

known

M : partially known

b, u, W, g, e : unknown

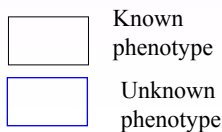
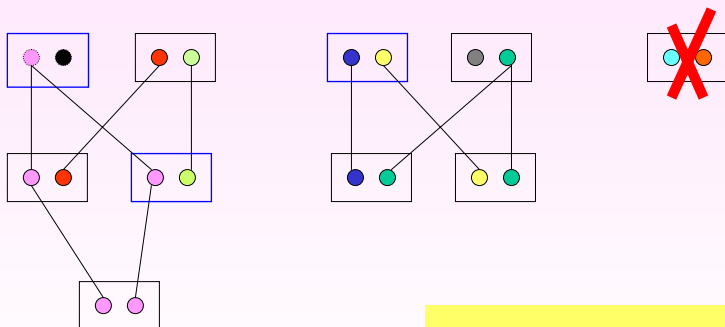
Rank W, g : unknown

Genic action : unknown

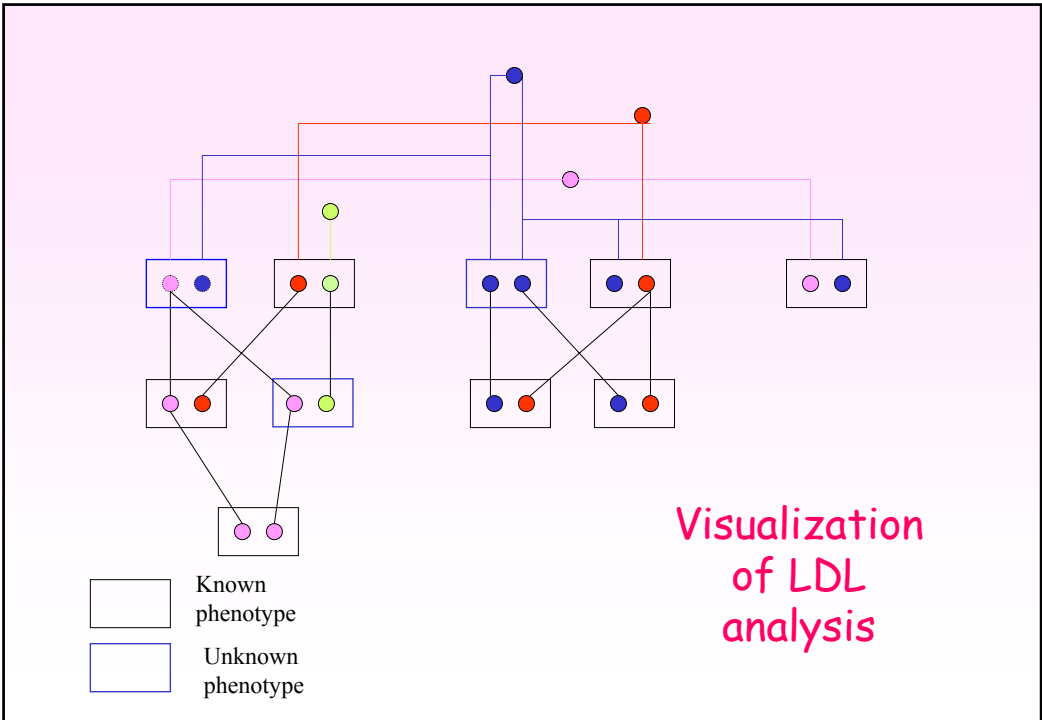
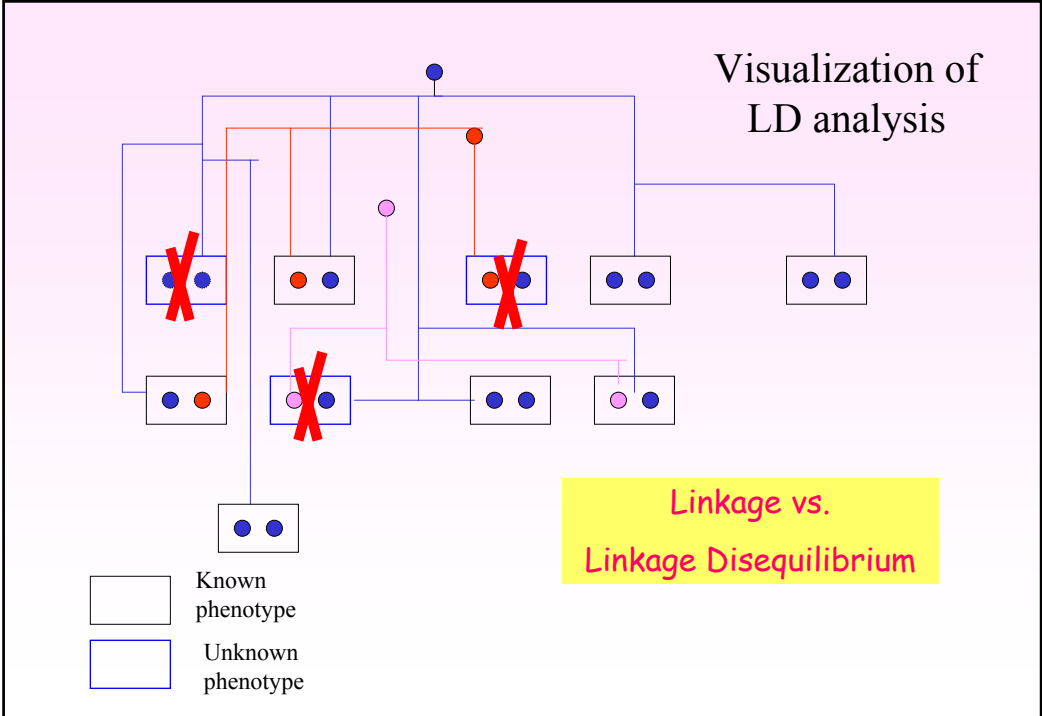
Day 1. Fine mapping and analysis of complex pedigrees

1. Combining linkage and linkage disequilibrium information
2. Analysis of crosses between outbred lines
3. QxPak software

Visualization of linkage analysis



Linkage vs
Linkage Disequilibrium



Combining linkage and LD

Linkage analysis:

- Robust, but not very accurate.
- Assumes all alleles from base population are different.

Linkage disequilibrium:

- It can be very accurate but very sensitive to departures from model assumptions.
- Risk of false positives.
- The region of maximum LD may not coincide with the QTL position.
- A pure LD analysis disregards pedigree structure.

Potential advantages of LDL

Homozygous genotypes contribute information.

Non related individuals with phenotype also.

Offspring phenotypes contribute to assess the likely genotype of their parents, thus making it more robust than simply LD.

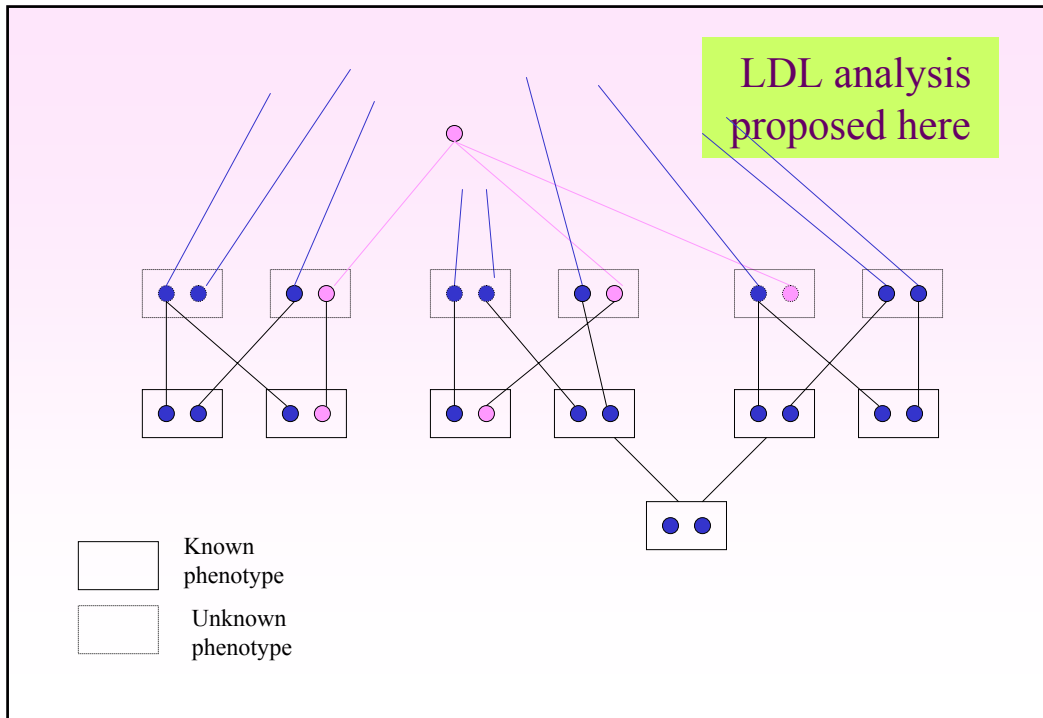
By comparing L, LD, and LDL estimates we can verify the assumptions in modelling LD decay.

LDL mapping

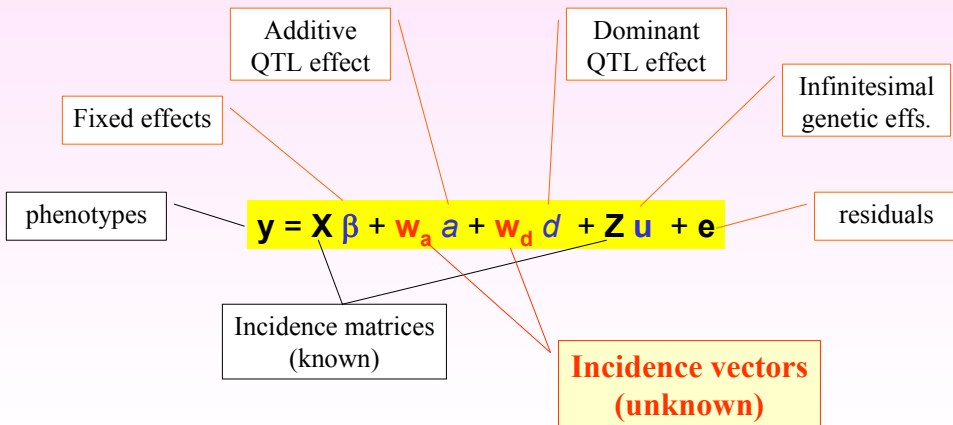
(Pérez-Enciso, Genetics, 2003)

Assumptions:

- QTL identified within a predetermined region.
- A biallelic QTL with a mutant allele appearing t generations ago on a single haplotype.
- A star shape (exponential growth) genealogy.

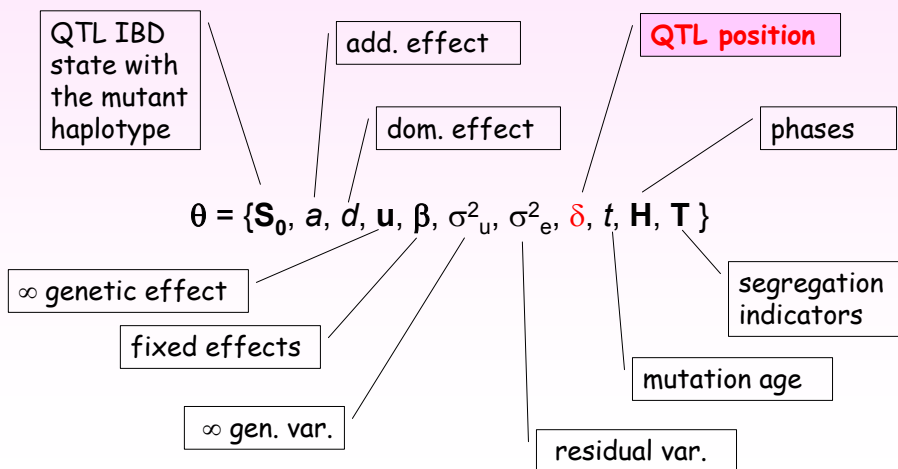


The model



Parameters to be estimated

$$y = X\beta + w_a a + w_d d + Z u + e$$



Bayesian inference

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{w}_a \mathbf{a} + \mathbf{w}_d \mathbf{d} + \mathbf{Z} \mathbf{u} + \mathbf{e}$$

Parameters to be estimated

$$\boldsymbol{\theta} = \{\mathbf{S}_0, \mathbf{a}, \mathbf{d}, \mathbf{u}, \boldsymbol{\beta}, \sigma_u^2, \sigma_e^2, \delta, t, \mathbf{H}, \mathbf{T}\}$$

Bayes posterior

$$p(\boldsymbol{\theta} | \mathbf{y}, \mathbf{M}) \propto p(\mathbf{y}, \mathbf{M} | \boldsymbol{\theta}) p(\boldsymbol{\theta}) = p(\mathbf{y} | \boldsymbol{\theta}) p(\mathbf{M} | \boldsymbol{\theta}) p(\boldsymbol{\theta})$$

Marginal Bayes posterior

$$p(\boldsymbol{\theta}_i | \mathbf{y}, \mathbf{M}) = \int_{\boldsymbol{\theta}_{-i}} p(\boldsymbol{\theta}_i, \boldsymbol{\theta}_{-i} | \mathbf{y}, \mathbf{M}) \partial \boldsymbol{\theta}_{-i}$$

Bayesian inference principles

(joint)
posterior

likelihood

prior

$$p(\boldsymbol{\theta} | \mathbf{y}) = \frac{p(\mathbf{y} | \boldsymbol{\theta}) p(\boldsymbol{\theta})}{p(\mathbf{y})}$$

data density

marginal
posterior

$$p(\boldsymbol{\theta}_i | \mathbf{y}, \mathbf{M}) = \int_{\boldsymbol{\theta}_{-i}} p(\boldsymbol{\theta}_i, \boldsymbol{\theta}_{-i} | \mathbf{y}, \mathbf{M}) \partial \boldsymbol{\theta}_{-i}$$

Implementation: Monte Carlo Markov Chain

1. $\theta_1 \sim p(\theta_1 | \theta_{-1}, \mathbf{y}, \mathbf{M})$
2. $\theta_2 \sim p(\theta_2 | \theta_{-2}, \mathbf{y}, \mathbf{M})$
3. $\theta_3 \sim p(\theta_3 | \theta_{-3}, \mathbf{y}, \mathbf{M})$
- .
- .
- .
- m. $\theta_m \sim p(\theta_m | \theta_{-m}, \mathbf{y}, \mathbf{M})$

Implementation: Monte Carlo Markov Chain

1. $\theta_1 \sim p(\theta_1 | \theta_{-1}, \mathbf{y}, \mathbf{M}) \rightarrow \theta_1 \sim p(\theta_1 | \mathbf{y}, \mathbf{M})$
2. $\theta_2 \sim p(\theta_2 | \theta_{-2}, \mathbf{y}, \mathbf{M}) \rightarrow \theta_2 \sim p(\theta_2 | \mathbf{y}, \mathbf{M})$
3. $\theta_3 \sim p(\theta_3 | \theta_{-3}, \mathbf{y}, \mathbf{M}) \rightarrow \theta_3 \sim p(\theta_3 | \mathbf{y}, \mathbf{M})$
- .
- .
- .
- m. $\theta_m \sim p(\theta_m | \theta_{-m}, \mathbf{y}, \mathbf{M}) \rightarrow \theta_m \sim p(\theta_m | \mathbf{y}, \mathbf{M})$

Priors assumed

Flat unbounded priors
 a, d, β

Flat bounded priors
 δ, t

Naive ignorance priors (hyperparameter $\nu = 0$)
 σ_u^2, σ_e^2

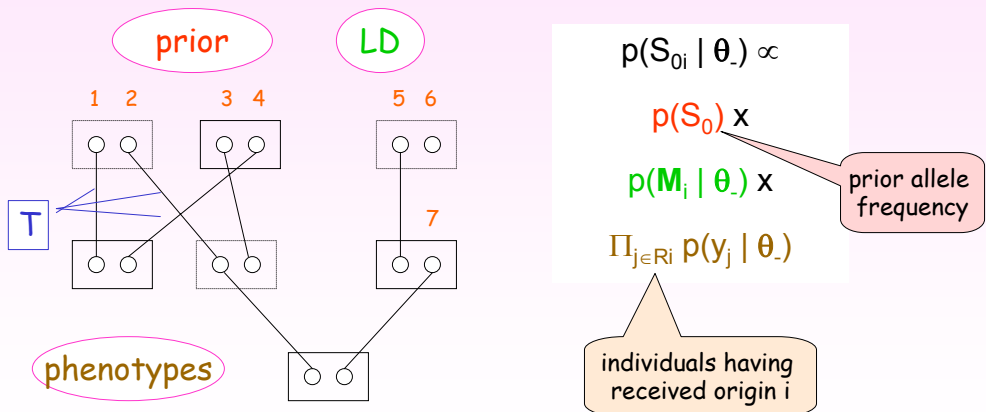
Binomial priors
 S_0 (prior QTL frequency), **H, T**

Multivariate normal
 $p(\mathbf{u}) = N(\mathbf{0}, \mathbf{A} \sigma_u^2)$

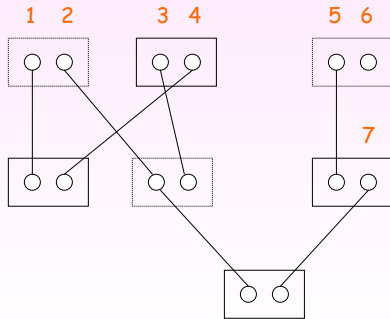
Transmission indicators

Phases

Sampling QTL alleles $(S_0 \mid a, d, \beta, u, T, H, M, y, t, \sigma_e^2)$



Contribution from phenotypes
 $(\mathbf{y} \mid S_0, \mathbf{a}, \mathbf{d}, \boldsymbol{\beta}, \mathbf{u}, \sigma_e^2)$

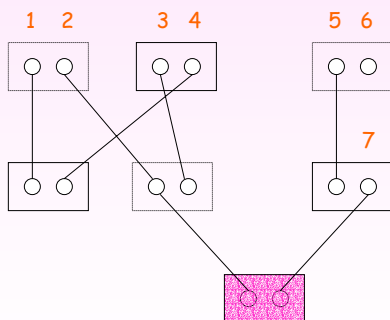


$$\prod_{j \in Ri} p(\mathbf{y}_j \mid \boldsymbol{\theta}_j)$$

$$p(\mathbf{y}_i \mid S_{0j}, \mathbf{a}, \mathbf{d}, \mathbf{u}_i, \boldsymbol{\beta}, \sigma_e^2) =$$

$$N(\mathbf{y}_j - \mathbf{x}_i' \boldsymbol{\beta} - \mathbf{u}_j - w_{aj} \mathbf{a} - w_{dj} \mathbf{d}, \sigma_e^2)$$

Contribution from phenotypes
 $(\mathbf{y} \mid S_0, \mathbf{a}, \mathbf{d}, \boldsymbol{\beta}, \mathbf{u}, \sigma_e^2)$



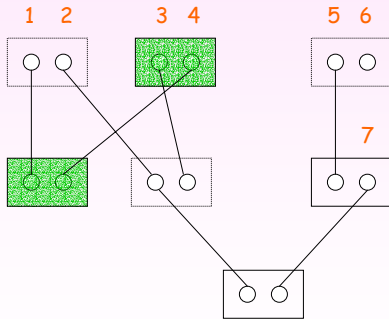
$$\prod_{j \in Ri} p(\mathbf{y}_j \mid \boldsymbol{\theta}_j)$$

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$$N(\mathbf{y}_j - \mathbf{x}_i' \boldsymbol{\beta} - \mathbf{u}_j - w_{aj} \mathbf{a} - w_{dj} \mathbf{d}, \sigma_e^2)$$

Example: origin 2

Contribution from phenotypes
 $(\mathbf{y} \mid S_0, \mathbf{a}, \mathbf{d}, \boldsymbol{\beta}, \mathbf{u}, \sigma_e^2)$



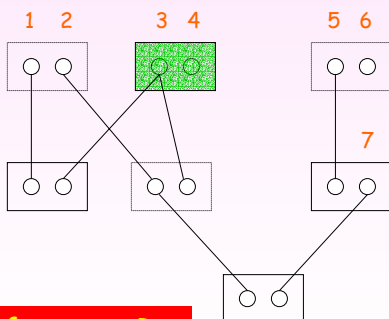
$$\prod_{j \in R_i} p(y_j \mid \theta_{\cdot})$$

$$p(y_i \mid S_{0j}, \mathbf{a}, \mathbf{d}, u_i, \boldsymbol{\beta}, \sigma_e^2) =$$

$$N(y_j - \mathbf{x}_i' \boldsymbol{\beta} - u_j - w_{aj} \mathbf{a} - w_{dj} \mathbf{d}, \sigma_e^2)$$

Example: origin 4

Contribution from phenotypes
 $(\mathbf{y} \mid S_0, \mathbf{a}, \mathbf{d}, \boldsymbol{\beta}, \mathbf{u}, \sigma_e^2)$



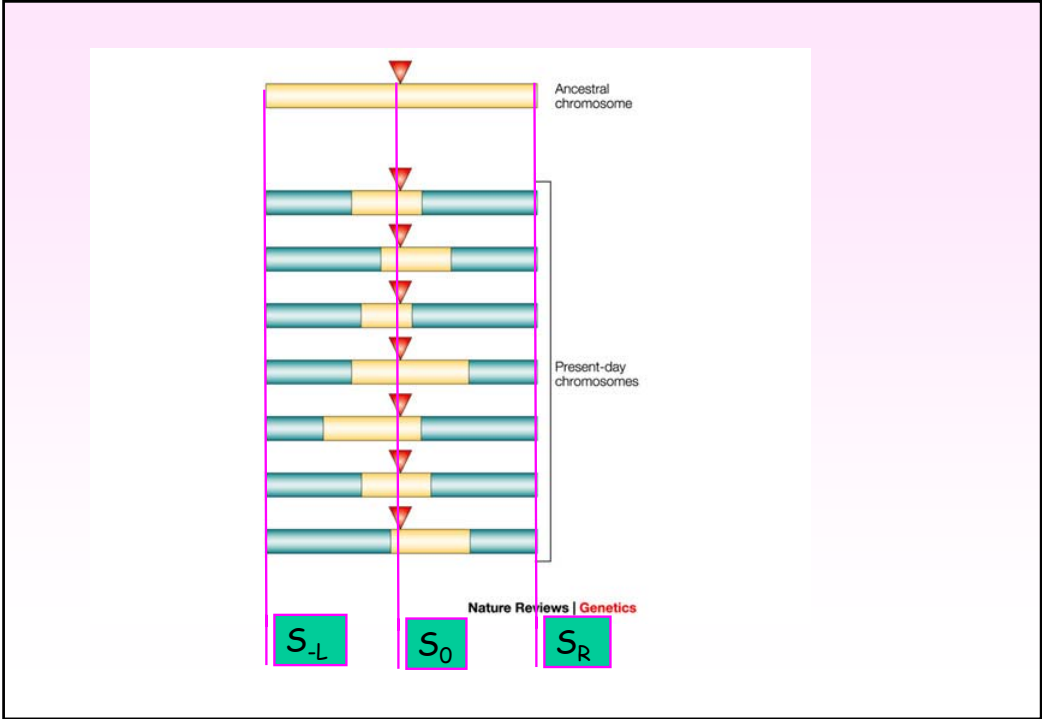
$$\prod_{j \in R_i} p(y_j \mid \theta_{\cdot})$$

$$p(y_i \mid S_{0j}, \mathbf{a}, \mathbf{d}, u_i, \boldsymbol{\beta}, \sigma_e^2) =$$

$$N(y_j - \mathbf{x}_i' \boldsymbol{\beta} - u_j - w_{aj} \mathbf{a} - w_{dj} \mathbf{d}, \sigma_e^2)$$

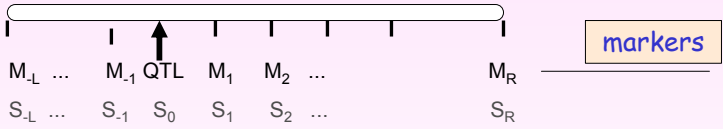
Of course R_i
 depends on $T!$

Example: origin 4



Contribution from disequilibrium: $p(M | \theta)$

Haplotype i



S_k is IBD indicator for position k
 $S_k = 1 \equiv k$ th marker allele is IBD with initial mutant haplotype
 $S_k = 0 \equiv k$ th marker allele not IBD with initial mutant haplotype
 $k = 0 \equiv$ QTL position

based in Morris et al. (2000)

Contribution from disequilibrium: $p(M | \theta)$

$$\begin{aligned}
 p(\mathbf{M}_i | S_{0i}, t, \mathbf{H}, \delta) &= \\
 &= p(M_{i-L}, \dots, M_{i-1}, M_{i1}, M_{i2}, \dots, M_{iR} | \theta) = \\
 &= p(M_{i-L}, \dots, M_{i-1} | \theta) p(M_{i1}, M_{i2}, \dots, M_{iR} | \theta) = Q_{iL} Q_{iR}
 \end{aligned}$$

'left' part

'right' part

$$\begin{aligned}
 Q_{iR} &= p(M_1, M_2, \dots, M_R | \theta) = \\
 &= \sum_{S_1=0,1} p(M_1, M_2, \dots, M_R | S_1, S_0, \theta_{-}) = \\
 &= \sum_{S_1=0,1} p(M_2, \dots, M_R | S_1, \theta_{-}) p(M_1 | S_1, \theta_{-}) p(S_1 | S_0, \theta_{-})
 \end{aligned}$$

remaining haplotype

prob. allele | IBD state

transition probabilities

Contribution from disequilibrium: $p(M | \theta)$

$$\begin{aligned}
 p(\mathbf{M}_i | S_{0i}, t, \mathbf{H}, \delta) &= \\
 &= p(M_{i-L}, \dots, M_{i-1}, M_{i1}, M_{i2}, \dots, M_{iR} | \theta) = \\
 &= p(M_{i-L}, \dots, M_{i-1} | \theta) p(M_{i1}, M_{i2}, \dots, M_{iR} | \theta) = Q_{iL} Q_{iR}
 \end{aligned}$$

'left' part

'right' part

$$\begin{aligned}
 Q_R &= p(M_1, M_2, \dots, M_R | S_0, \dots) = \\
 &= \sum_{S_1} p(M_2, \dots, M_R | S_1) p(M_1 | S_1) p(S_1 | S_0) = \\
 &= \sum_{S_1} \sum_{S_2} p(M_3, \dots, M_R | S_2) \underbrace{p(M_2 | S_2)} \underbrace{p(S_2 | S_1)} \underbrace{p(M_1 | S_1)} \underbrace{p(S_1 | S_0)} = \\
 &= \sum_{S_1} \sum_{S_2} \dots \sum_{S_R} \prod_{k=1}^R p(M_k | S_k) p(S_k | S_{k-1})
 \end{aligned}$$

S_k are integrated out

Contribution from disequilibrium: $p(M | \theta)$

Marker allele
probs | IBD state

$$Q_R = p(M_1, M_2, \dots, M_R | \theta) = \sum_{S_1} \sum_{S_2} \dots \sum_{S_R} \prod_{k=1}^R p(M_k | S_k) p(S_k | S_{k-1})$$

transition
IBD prob.

$p(M_k | S_k = +)$ is simply the population allele frequencies
 $p(M_k | S_k = -) = 1$ if that allele was carried by the mutant haplotype
 $p(M_k | S_k = -) = 0$ for the remaining alleles

t , time since mutation

r , recombination rate btw markers

$$p(S_k = - | S_{k-1} = -) = \exp(-t r_{k,k+1}) + [1 - \exp(-t r_{k,k+1})] \alpha$$

$$p(S_k = - | S_{k-1} = +) = [1 - \exp(-t r_{k,k+1})] \alpha$$

$$p(S_k = + | S_{k-1} = -) = [1 - \exp(-t r_{k,k+1})] (1 - \alpha)$$

$$p(S_k = + | S_{k-1} = +) = \exp(-t r_{k,k+1}) + [1 - \exp(-t r_{k,k+1})] (1 - \alpha)$$

α , prob of recombining with a
haplotype carrying the mutation

Morris et al. (2000)

Rearranging...

$$Q_R = p(M_1, M_2, \dots, M_R | \theta) = \sum_{S_1} \sum_{S_2} \dots \sum_{S_R} \prod_{k=1}^R p(M_k | S_k) p(S_k | S_{k-1})$$

$$= \sum_{S_1} p(M_1 | S_1) p(S_1 | S_0) \dots \sum_{S_{R-1}} p(M_{R-1} | S_{R-1}) p(S_{R-1} | S_{R-2}) \sum_{S_R} p(M_R | S_R) p(S_R | S_{R-1})$$

Then:

$$Q_R = \sum_{k=R}^1 q_k$$

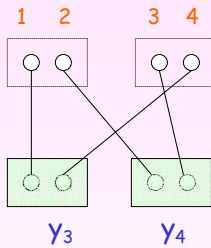
$$Q_L = \sum_{k=-L}^1 q_k$$

Where

$$q_k = \sum_{S_k} p(M_k | S_k) p(S_k | S_{k-1}) q_{k-1}$$

$$q_R = q_{-L} = 1$$

Block sampling QTL alleles



$$p(S_{01}, S_{02} | \theta_{\cdot}) \propto$$

$$p(S_0) \times$$

$$p(M_i | \theta_{\cdot}) \times$$

$$\prod_{j \in R_i} p(y_j | \theta_{\cdot})$$

S_{01}	S_{02}	$p(S_0)$	$p(M \theta)$	$p(y \theta)$
+	+	$\propto p(S_0=+)^2$	$p(M_1 S_0=+)p(M_2 S_0=+)$	$p(y_3 S_0=+)p(y_4 S_0=+)$
+	-	$\propto p(S_0=+)p(S_0=-)$	$p(M_1 S_0=+)p(M_2 S_0=-)$	$p(y_3 S_0=+)p(y_4 S_0=-)$
-	+	$\propto p(S_0=+)p(S_0=-)$	$p(M_1 S_0=-)p(M_2 S_0=+)$	$p(y_3 S_0=-)p(y_4 S_0=+)$
-	-	$\propto p(S_0=-)^2$	$p(M_1 S_0=-)p(M_2 S_0=-)$	$p(y_3 S_0=-)p(y_4 S_0=-)$

Sampling mixed model effects (α , d , β , u)

Conditional on w_a , w_d , and variances

$$\beta^* = (\beta', a, d)'$$

$$X^* = (X, w_a, w_d)$$

$$\begin{bmatrix} X^{*'} X^* & X^{*'} Z \\ Z' X^* & Z' Z + A^{-1} \lambda \end{bmatrix} \begin{bmatrix} \beta^* \\ u \end{bmatrix} = \begin{bmatrix} X^{*'} y \\ Z y \end{bmatrix} \equiv C b = r$$

$$\lambda = \sigma_e^2 / \sigma_u^2$$

$$(b_i | \theta_{\cdot}, y, M) \sim N\left(r_i - \sum_{j=1, j \neq i}^{\text{rank}(C)} c_{ij} r_j, \sigma_e^2 / c_{ii}\right), \quad \forall i \leftarrow \text{Normal } f(x)$$

any of β , u , a , or d parameters

Sampling variances

$$p(\sigma_u^2 | S_0, a, d, \mathbf{u}, \boldsymbol{\beta}, \sigma_e^2, \mathbf{y}) = (\mathbf{u}' \mathbf{A}^{-1} \mathbf{u}) \chi_m^{-2}$$

$$p(\sigma_e^2 | S_0, a, d, \mathbf{u}, \boldsymbol{\beta}, \sigma_u^2, \mathbf{y}) =$$

$$= (\mathbf{y} - \mathbf{X}^* \boldsymbol{\beta}^* - \mathbf{Z} \mathbf{u})' (\mathbf{y} - \mathbf{X}^* \boldsymbol{\beta}^* - \mathbf{Z} \mathbf{u}) \chi_n^{-2}$$

Chi squared

LD parameters: t

Metropolis Hastings

The new proposed t_{new} is accepted with probability:

$$\min \left\{ 1, \frac{p(\mathbf{M} | \mathbf{S}_0, t^{\text{new}}, \mathbf{H}, \delta)}{p(\mathbf{M} | \mathbf{S}_0, t, \mathbf{H}, \delta)} \right\}$$

LD parameters: $p(M|S)$

Marker non IBD with founder mutant haplotype

$$p(M_{kj} | S_{k=+}, \theta_-) = \sum_{i=1}^{N_b} \sum_{h=1}^2 p(S_{kih} = + | S_{0ih}) \eta_{ihjk} / (2 N_b)$$

Marker IBD status
 Marker k, allele j
 ith indiv among base popn.
 h-th haplotype
 # indivs. in base popn.
 = 1 if i indiv, h hap, has k allele at j marker ; 0 otherwise

Marker IBD with founder mutant haplotype

$$p(M_{kj} | S_{k=-}, \theta_-) = \sum_{i=1}^{N_b} \sum_{h=1}^2 p(S_{kih} = - | S_{0ih}) \eta_{ihjk} / (2 N_b)$$

Phases, H

are sampled in blocks of n_h phases of the same individual jointly via Gibbs sampling (see above this chapter).

Segregation indicators, T

are sampled jointly with QTL position following Mendelian rules and using available marker information.

QTL position { δ }

Sampling δ is, together with updating QTL alleles, the most critical aspect of QTL Bayesian implementation. This occurs because \mathbf{S}_0 , \mathbf{T} , \mathbf{H} and δ are highly interdependent and it is difficult to update them all simultaneously.

Conditional on \mathbf{S}_0 and \mathbf{T} , updating δ is a straightforward, and it is like a standard linkage analysis.

But this simple approach is very prone to get δ stuck within a marker bracket because, conditional on a given set of crossovers, it is very unlikely to 'jump' to the next marker bracket.

A mixed approach was followed here
(Uimari and Sillanpaa, 2001)

A new δ is accepted with prob.

$$\text{Min} \left\{ 1, \frac{p(\mathbf{T} | \delta^{\text{new}}, \mathbf{H})}{p(\mathbf{T} | \delta, \mathbf{H})} \right\}$$

This ratio depends on the crossovers that have occurred when the QTL is assumed to be in position δ or in δ^{new}

or, every \tilde{n} iterations, with prob.

$$\text{Min} \left\{ 1, \frac{p(\mathbf{y} | \mathbf{T}^{\text{new}}, \mathbf{S}_0, \mathbf{a}, \mathbf{d}, \mathbf{u}, \boldsymbol{\beta}, \sigma_e^2)}{p(\mathbf{y} | \mathbf{T}, \mathbf{S}_0, \mathbf{a}, \mathbf{d}, \mathbf{u}, \boldsymbol{\beta}, \sigma_e^2)} \right\}$$

This ratio is computed after sampling a new \mathbf{T} set conditional on δ^{new} , the ratio depends on how fit new genotypes to observed phenotypes

Example: Simulation study

‘Simple’ pedigree (n = 480):

40 fullsib families; 10 offspring / family

‘Complex’ pedigree (n = 480)

4 generation pedigree; 80 parents; 5 fullsibs / family

Region explored = 25 cM

6 microsatellites and 11 SNPs

Additive effect = 1

Dominant eff = 0

Residual var = 1

Complete association

All haps with $\text{SNP}_{18} = 2$ had mutant QTL allele

Incomplete association

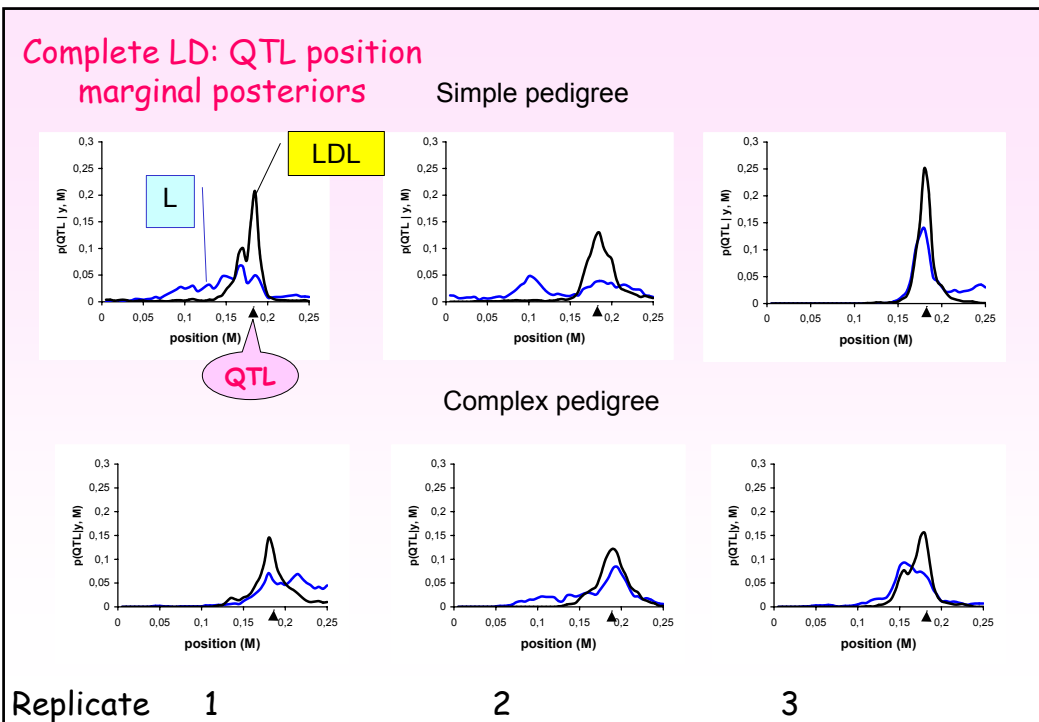
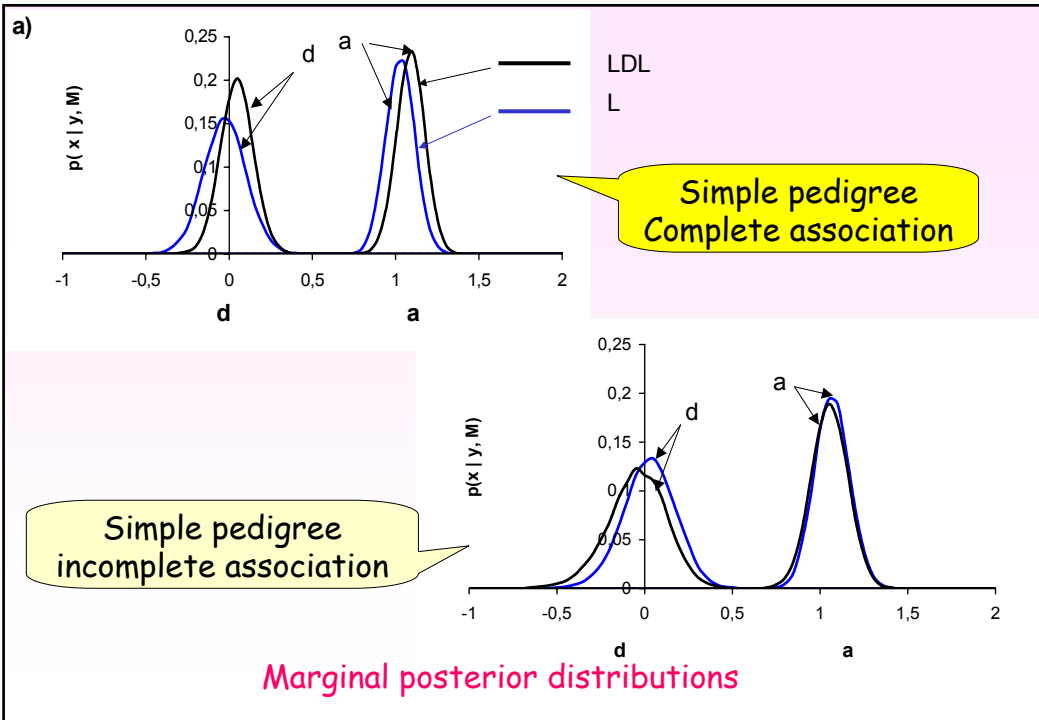
Initially, 42% of had $\text{SNP}_{18} = 1$ had mutant QTL allele

In all cases, star shape genealogy

Main results

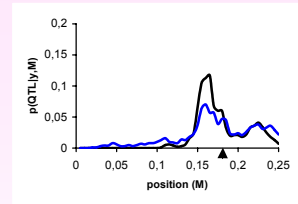
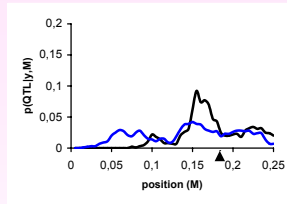
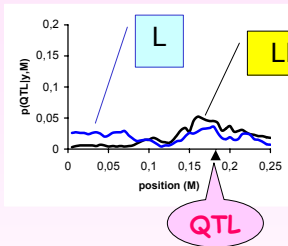
	Popn.	Method	$E(\hat{a} y)$	$E(\hat{d} y)$	$E(\hat{\delta} y)$	$\text{Var}(\hat{\delta} y)^{0.5}$
Complete association	Simple	LDL	1.07	0.06	0.177	0.024
		L	1.04	0.03	0.161	0.045
	Complex	LDL	0.96	0.00	0.179	0.020
		L	0.91	0.02	0.175	0.035
Incomplete assoc.	Simple	LDL	1.03	-0.08	0.170	0.041
		L	1.04	-0.01	0.144	0.060
	Complex	LDL	0.88	0.06	0.185	0.026
		L	0.89	0.10	0.180	0.032
	True		1.00	0.00	0.18	

Average of 3 replicates

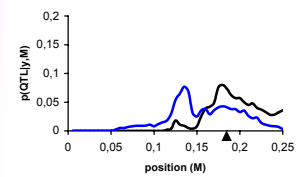
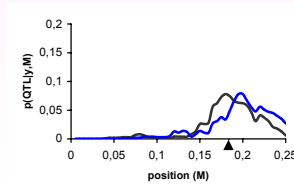
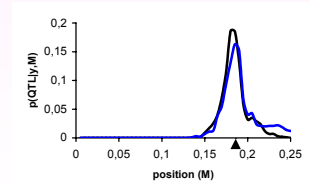


Incomplete LD: QTL position marginal posteriors

Simple pedigree



Complex pedigree



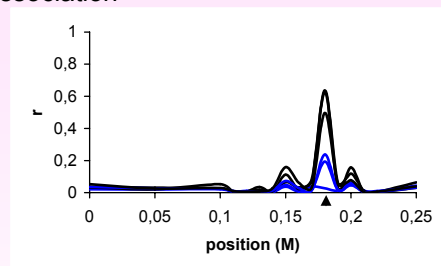
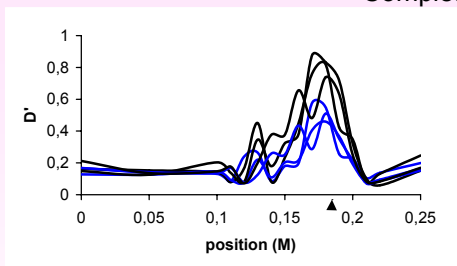
Replicate 1

2

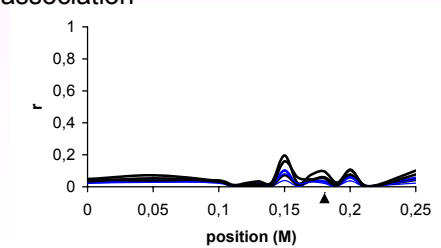
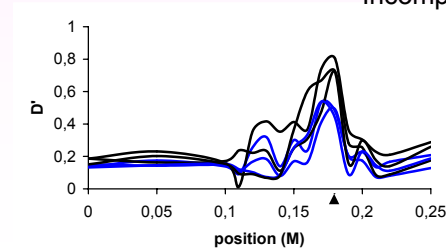
3

Disequilibrium measures: simple pedigree

Complete association



Incomplete association



Conclusions

The advantage of LDL over linkage only will depend on the structure of the population as well as on the validity of the LD model.

Uncertainty on phases and on QTL alleles makes it LDL to perform much poorer than expected.


It seems that LDL increase in accuracy should not be overestimated.

A very exciting and timely area of research, many open fronts and approaches.

Other approaches

Allison, D. B., Heo, M., Kaplan, N., & Martin, E. R. (1999). Sibling-based tests of linkage and association for quantitative traits. *Am J Hum Genet* **64**, 1754-1763.

Farnir, F., Grisart, B., Coppieters, W., Riquet, J., Berzi, P., Cambisano, N., Karim, L., Mni, M., Moisiso, S., Simon, P., Wagenaar, D., Vilkki, J., & Georges, M. (2002). Simultaneous Mining of Linkage and Linkage Disequilibrium to Fine Map Quantitative Trait Loci in Outbred Half-Sib Pedigrees. *Genetics* **161**, 275-287.

 Meuwissen, T. H., Karlsen, A., Lien, S., Olsaker, I., & Goddard, M. E. (2002). Fine mapping of a quantitative trait locus for twinning rate using combined linkage and linkage disequilibrium mapping. *Genetics* **161**, 373-379.

Meuwissen & Goddard's approach

The goal is to compute the probabilities that two haplotypes are identical by descent (IBD) at a given position or segment

For any given position $G=\{g_{ij}\}$ contains these P_{IBD}

G is later used in a maximum likelihood approach

Meuwissen & Goddard's approach

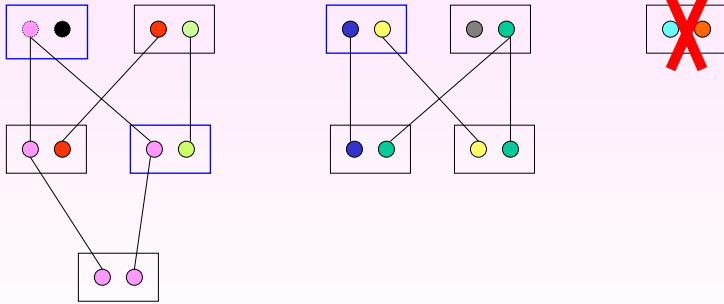
In a usual analysis, base population individuals are assumed to be unrelated.



But if we use LD, this is no longer true.

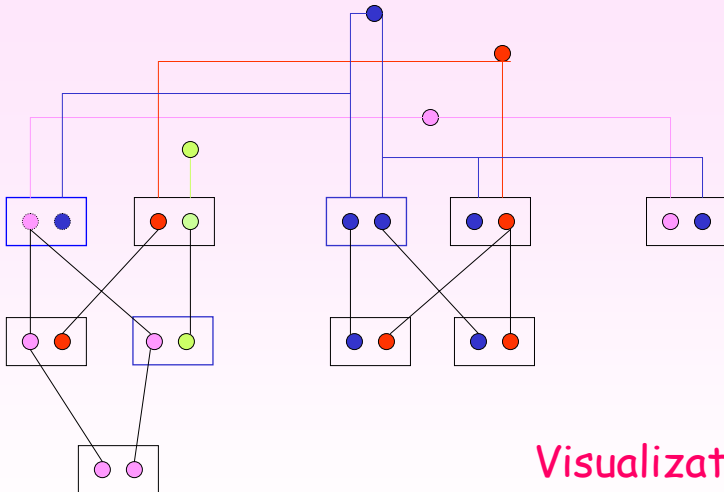
M&G present a method to compute the relationship (IBD probs) between base population individuals.



The usual relationship between descendants is increased according to this base IBD probs.

Visualization of linkage analysis



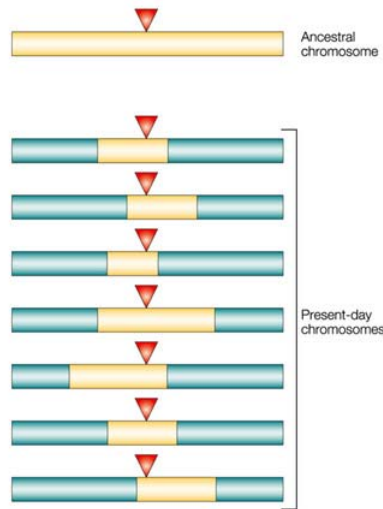
 Known phenotype
 Unknown phenotype



 Known phenotype
 Unknown phenotype

Visualization
of LDL
analysis

Haplotype erosion



Nature Reviews | Genetics

Ardlie et al. 2002

Prob. of two individuals sharing an intact chromosome segment

Depends on:

$c \equiv$ chr. length (\downarrow)

$t \equiv$ time in generations since most recent common ancestor (MRCA) (\downarrow)

$N_e \equiv$ effective size (\downarrow)

Prob. of two individuals sharing an intact chromosome segment

P MRCA in gen. t

P no common ancestor gens $t-1$

P no recombination in $2t$ gens.

$$\frac{1}{2N_e} \left(1 - \frac{1}{2N_e}\right)^{t-1} [\exp(-c)]^{2t}$$

$$\approx \frac{1}{2N_e} \exp\left[-\frac{t-1}{2N_e} - 2ct\right]$$

Prob. of two individuals sharing an intact chromosome segment of length c

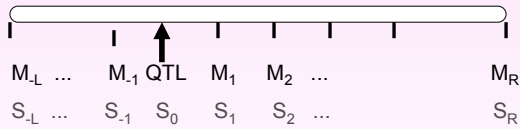
reference time

$$P_c = \frac{\exp(-2c)}{2N_e} \sum_{t=1}^T \exp\left[-(t-1)\left(\frac{1}{2N_e} + 2c\right)\right] =$$

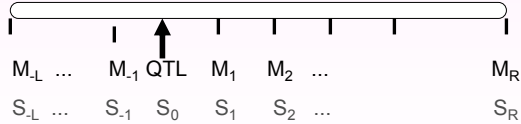
$$\frac{\exp(-2c)}{2N_e} \frac{1 - \exp\left[-T\left(2c + \frac{1}{2N_e}\right)\right]}{1 - \exp\left[-\left(2c + \frac{1}{2N_e}\right)\right]}$$

Prob. of two individuals being IBD

hap. i

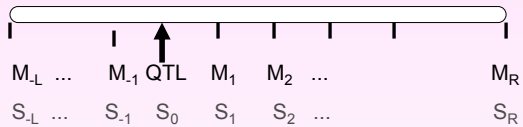


hap. j



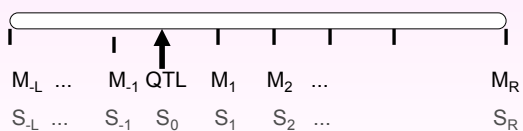
Prob. of two individuals being IBD

hap. i



identity by state (IBS) statuses

hap. j



IBD statuses

$$\phi \equiv \begin{cases} 1 & \dots & 0 & 1 & 1 & 1 & \dots \\ \dots & & x & x & _ & & \dots \end{cases}$$

recomb. events

M&G's original idea

Goal:

$$P(S_0 | M_1) = \frac{P(S_0, M_1)}{P(M_1)}$$

$$= \frac{\sum_{S_1} P(M_1 | S_{0,1}) P(S_{0,1})}{P(M_1)}$$

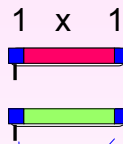
S_0	S_1	M_1	$p(M S)$	$P(S)$
0	0	0	$1-p^2$	
1	0	0	$1-p^2$	
0	1	1	1	
0	0	1	p^2	
1	1	1	1	P_c
1	1	1	1	
1	0	1	p^2	

see eqn. before

freq. ²
allele
marker 1

M&G's original idea

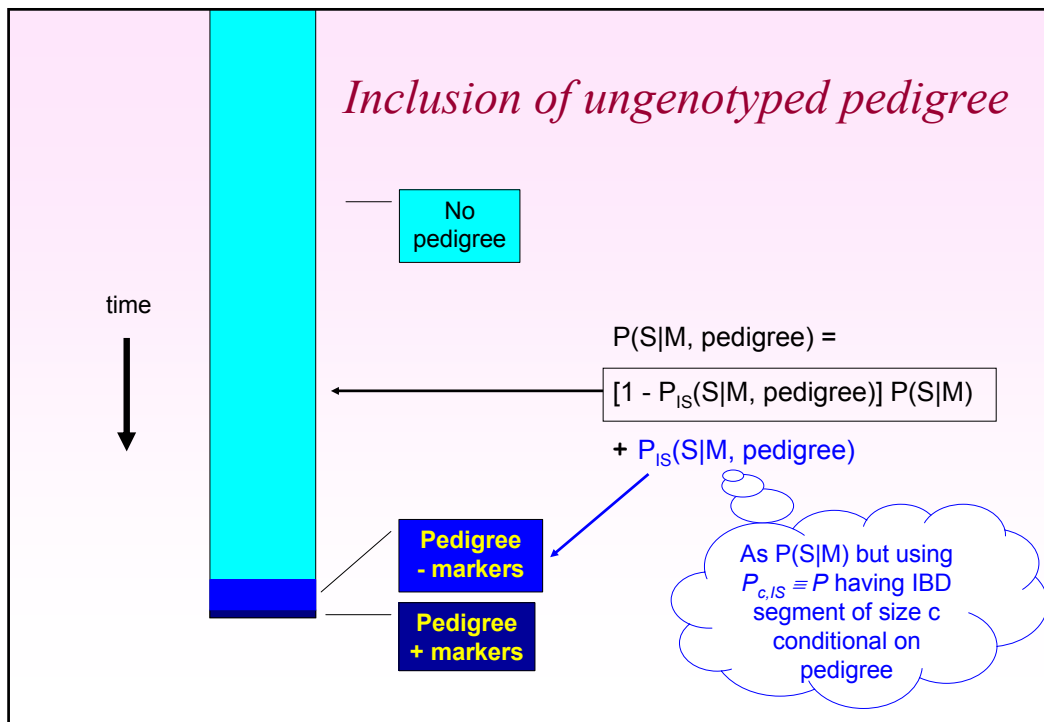
S_0	S_1	M_1	$p(M S)$	$P(S)$
0	0	0	$1-p^2$	
1	0	0	$1-p^2$	
0	1	1	1	
0	0	1	p^2	
1	1	1	1	P_c
1	1	1	1	?
1	0	1	p^2	



two regions of
size 0
bracketing a
region of size c

$$P('1x1') = P_0 (1-P_c) P_0 = E(F)^2 (1-P_c)$$

$$1 - \exp(-T/2Ne) = E(F)$$



What do we do next?

1. In the end, we obtain $P(\text{IBD}|M)$ at any desired genome positions
2. ML estimates can be obtained maximizing

$$\ln L = -1/2 [\text{Constant} + \log|V| + (\mathbf{y} - \mathbf{X} \mathbf{b})' \mathbf{V}^{-1} (\mathbf{y} - \mathbf{X} \mathbf{b})],$$

with

$$\mathbf{V} = \mathbf{Z} \mathbf{G} \mathbf{Z}' + \mathbf{R}$$

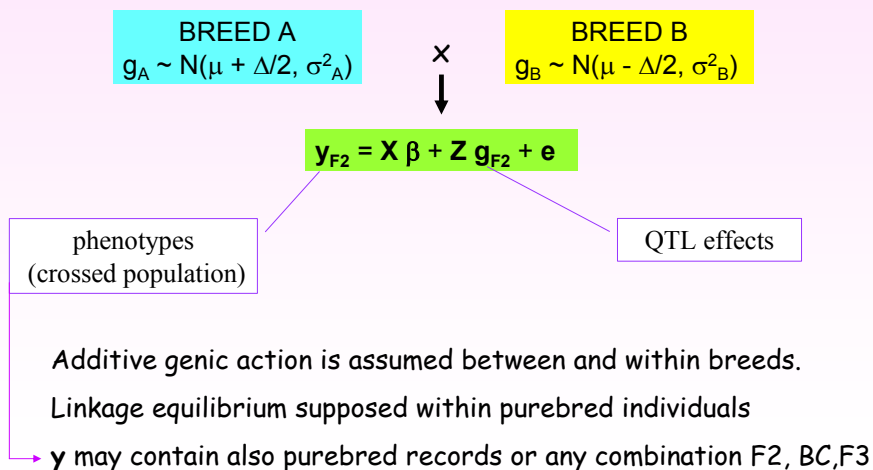
where

$$\mathbf{G} = \{ P_{ij}(\text{IBD}|M) \}$$

Day 1. Fine mapping and analysis of complex pedigrees

1. Combining linkage and linkage disequilibrium information
2. Analysis of crosses between outbred lines
3. QxPak software

Analysis of crosses between outbred lines Pérez-Enciso & Varona (2000)



$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{g} + \mathbf{e}$$

$$\begin{pmatrix} \mathbf{y} \\ \mathbf{g} \\ \mathbf{e} \end{pmatrix} \sim N \left[\begin{pmatrix} \mathbf{X}\boldsymbol{\beta} + \mathbf{P}\boldsymbol{\Delta} \\ \mathbf{P}\boldsymbol{\Delta} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{V} & \mathbf{GZ}' & \mathbf{R} \\ \mathbf{ZG} & \mathbf{G} & \mathbf{0} \\ \mathbf{R} & \mathbf{0} & \mathbf{R} \end{pmatrix} \right]$$

prob. of indiv i having received and allele of breed A origin at QTL j

$$\mathbf{P} = \{ (p_{ij} - 1/2) \}$$

$$\boldsymbol{\Delta} = \{ \Delta_j \}$$

j -th allelic effect

$$\mathbf{V} = \mathbf{ZGZ}' + \mathbf{R}$$

$$\mathbf{G} = \sum \mathbf{G}_j; \mathbf{G}_j = \{ \text{Cov}(g_{ij}, g_{ij}) \}$$

$$\mathbf{R} = \mathbf{I}\sigma_e^2$$

j -th QTL covariance (IBD) matrix

Define an indicator variable

$w = AA, AB, BA, BB$, depending on locus origin

$$\text{Var}(g_i) = \sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ E \left[\text{Cov}(g_{ij}^h, g_{ij'}^h | w_{jj'}) \right] + \text{Cov}_w \left[E(g_{ij}^h | w_{jj'}), E(g_{ij'}^h | w_{jj'}) \right] \right\}$$

Define an indicator variable

w = AA, AB, BA, BB, depending on locus origin

$$\text{Var}(g_i) = \sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ E \left[\text{Cov}(g_{ij}^h, g_{ij'}^h | w_{jj'}) \right] \right\} \text{Cov}_w \left[E(g_{ij}^h | w_{jj'}), E(g_{ij'}^h | w_{jj'}) \right] \right\}$$

$$\sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ E \left[\text{Cov}(g_{ij}^h, g_{ij'}^h | w_{jj'}) \right] \right\} = \begin{cases} 0 & \text{if } j \neq j' \\ 0 & \text{if } w = \text{AB or BA} \\ p_i \sigma_A^2 + (1 - p_i) \sigma_B^2 & \end{cases}$$

Define an indicator variable

w = AA, AB, BA, BB, depending on locus origin

$$\text{Var}(g_i) = \sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ E \left[\text{Cov}(g_{ij}^h, g_{ij'}^h | w_{jj'}) \right] \right\} \text{Cov}_w \left[E(g_{ij}^h | w_{jj'}), E(g_{ij'}^h | w_{jj'}) \right] \right\}$$

$$\sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ E \left[\text{Cov}(g_{ij}^h, g_{ij'}^h | w_{jj'}) \right] \right\} = \begin{cases} 0 & \text{if } j \neq j' \\ 0 & \text{if } w = \text{AB or BA} \\ p_i \sigma_A^2 + (1 - p_i) \sigma_B^2 & \end{cases}$$

$$\sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \sum_{j'=1}^{n_{\text{loci}}} \left\{ \text{Cov}_w \left[E(g_{ij}^h | w_{jj'}), E(g_{ij'}^h | w_{jj'}) \right] \right\} = f(r_{jj'}, \Delta, p_j^h, p_{j'}^h) \rightarrow 0$$

The **second term** in $\text{Var}(g)$ is the increased variance due to segregation in crossed individuals but note that it tends to zero **CONDITIONAL** on marker information if these are highly informative and closely spaced. Suppose we could isolate a set of F2 individuals whose genome origin could be known without error, its genetic variance would be exactly

$$\sum_{j=1}^{n_{\text{loci}}} \delta_j \sigma_{A_j}^2 + (1 - \delta_j) \sigma_{B_j}^2 ; \delta = \begin{cases} 1 & \text{if A origin} \\ 0 & \text{if B origin} \end{cases}$$

Then

$$\text{Var}(g_i) \cong \sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} p_{ij}^h \sigma_{A_j}^2 + (1 - p_{ij}^h) \sigma_{B_j}^2$$

Prob. of allele being of origin A

Can be applied to a QTL, or a genome portion, chr, etc

$$\text{Cov}(g_i, g_{i'}) = \sum_{h=1}^2 \sum_{j=1}^{n_{\text{loci}}} \left[\rho_{A(i,i')j}^h \sigma_{A_j}^2 + \rho_{B(i,i')j}^h \sigma_{B_j}^2 \right]$$

Prob. of alleles from both indivs. Being IBD and being of origin A (B)

Finally, as usual

ML estimates can be obtained maximizing

$$\ln L = -1/2 [\text{Constant} + \log|\mathbf{V}| + (\mathbf{y}-\mathbf{X}\beta)' \mathbf{V}^{-1} (\mathbf{y}-\mathbf{X}\beta)].$$

NOTE: This is a linearized likelihood in the sense that it approximates a mixture by a multivariate normal $\mathbf{y} \sim N(\mathbf{X}\beta, \mathbf{V})$.

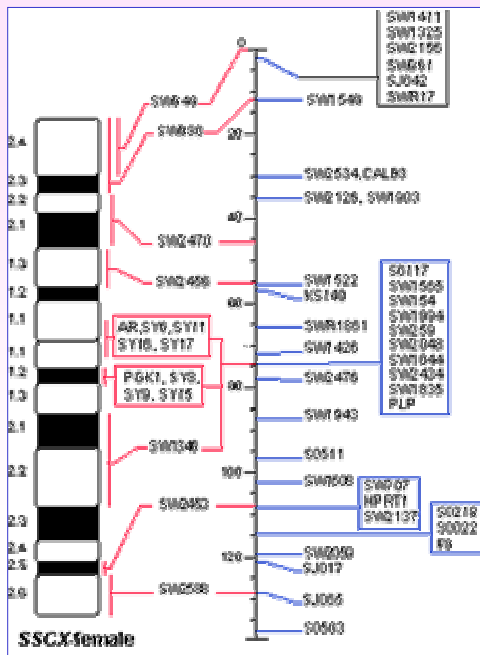
Examples:

Sex chromosome QTL in the IBMAP cross

Pérez-Enciso et al. (2002)

Whole genome analysis

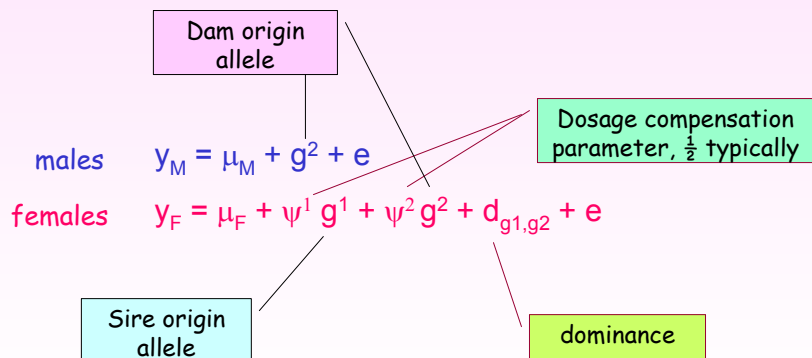
Ponz et al. (2001)



Porcine X chr

Recall:
 Different X/Y chr. lengths
 Dosage compensation in mammals

Dosage compensation modelling



Dosage compensation modelling

$E(g)$

$$\Pr(g_i^2 \in A)\mu_{gA} + \Pr(g_i^2 \in B)\mu_{gB} \text{ male}$$

$$\sum_{h=1}^2 \psi^h [\Pr(g_i^h \in A) \mu_{gA} + \Pr(g_i^h \in B) \mu_{gB}], \text{ females}$$

$\text{Cov}(g_i, g_{i'}) =$

$$\Pr(g_i^2 \equiv g_{i'}^2 \in A)\sigma_{Ag}^2 + \Pr(g_i^2 \equiv g_{i'}^2 \in B)\sigma_{Bg}^2 \text{ i, i' males}$$

Male
female

$$\sum_{h=1}^2 \psi^h [\Pr(g_i^h \equiv g_{i'}^h \in A) \sigma_{Ag}^2 + \Pr(g_i^h \equiv g_{i'}^h \in B) \sigma_{Bg}^2]$$

Female,
female

$$\sum_{h=1}^2 \sum_{h'=1}^2 \psi^h \psi^{h'} [\Pr(g_i^h \equiv g_{i'}^{h'} \in A) \sigma_{Ag}^2 + \Pr(g_i^h \equiv g_{i'}^{h'} \in B) \sigma_{Bg}^2]$$

Biometrical consequences of dosage compensation

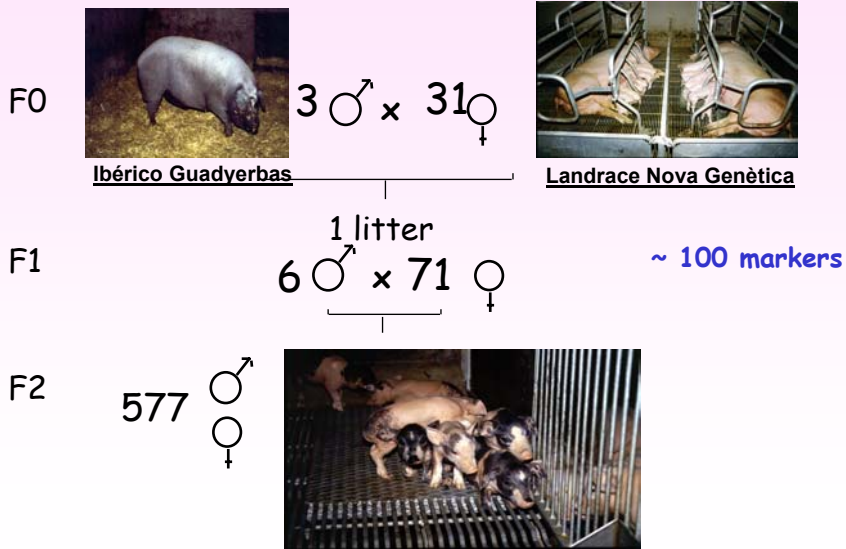
Provided that $\psi = 1/2$

$$\sigma_{gF}^2 = \frac{1}{2} \sigma_{gM}^2$$

$$E[\text{Cov}(FS_F)] = \frac{3}{4} \sigma_g^2; \quad \left[\frac{1}{2} \sigma_g^2 < \text{Cov} < \sigma_g^2 \right]$$

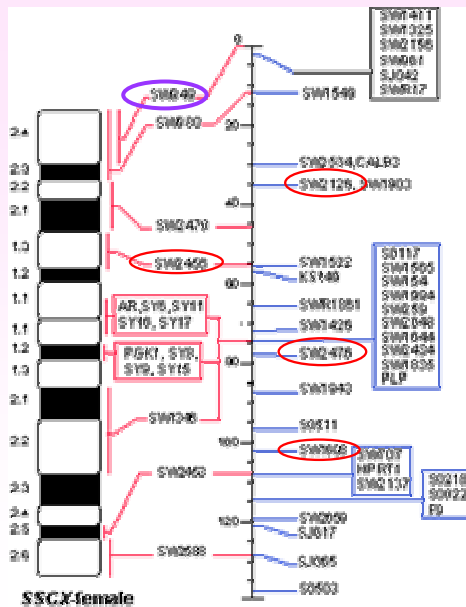
$$E[\text{Cov}(FS_{F,M})] = \frac{1}{4} \sigma_g^2; \quad \left[0 < \text{Cov} < \frac{1}{2} \sigma_g^2 \right]$$

IBMAP experimental protocol



Traits analyzed

- Carcass weight (**CW**)
- Carcass length (**CL**),
- pH at 24h post mortem (**pH**),
- Minolta meat color components, **a***, **b***, and **L***
- Haematin content (**Haem**)
- Subcutaneous backfat thickness (**BFT**)
- Longissimus muscle thickness (**LT**)
- Intramuscular fat percentage (**IMF**)



Markers typed

Statistical analysis strategy

Step 1: 5 cM segments

$$\text{Model - c1 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{c}_s \mathbf{a}_s + \mathbf{u}_0 + \mathbf{e}$$

$$\text{Model - v1 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{u}_s + \mathbf{u}_0 + \mathbf{e}$$

Step 2: 2cM segments

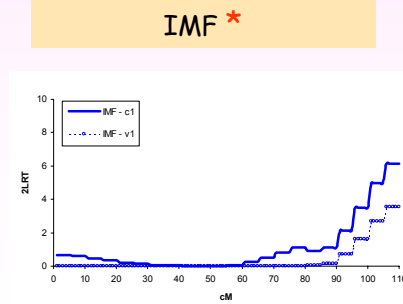
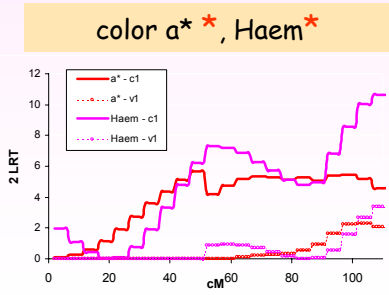
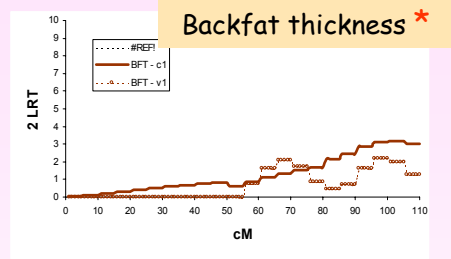
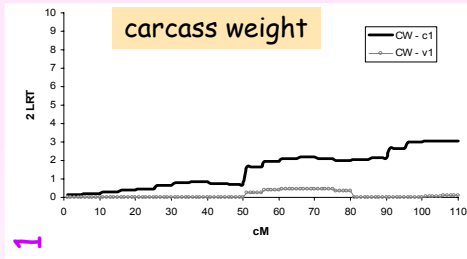
$$\text{Model - c1 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{c}_s \mathbf{a}_s + \mathbf{u}_0 + \mathbf{e}$$

$$\text{Model - c2 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{c}_s \mathbf{a}_s + \mathbf{c}'_s \mathbf{d}_s + \mathbf{u}_0 + \mathbf{e}$$

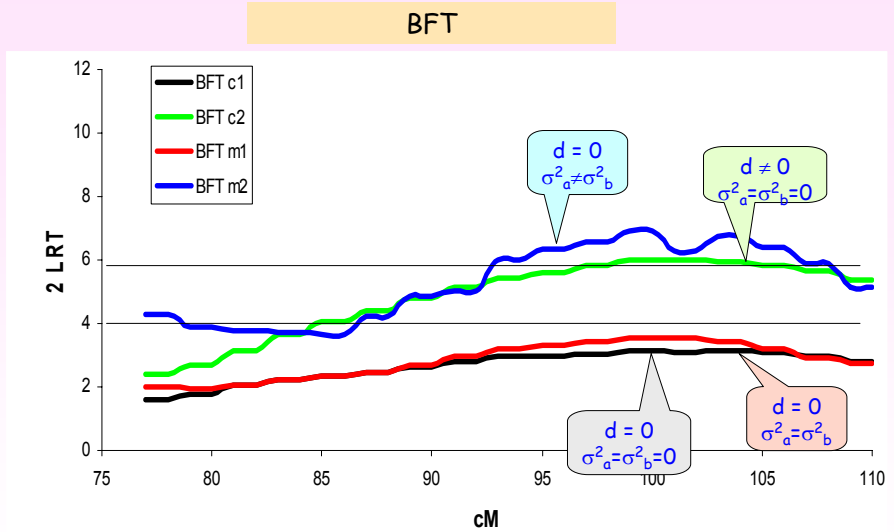
$$\text{Model - m1 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{c}_s \mathbf{a}_s + \mathbf{u}_s + \mathbf{u}_0 + \mathbf{e}$$

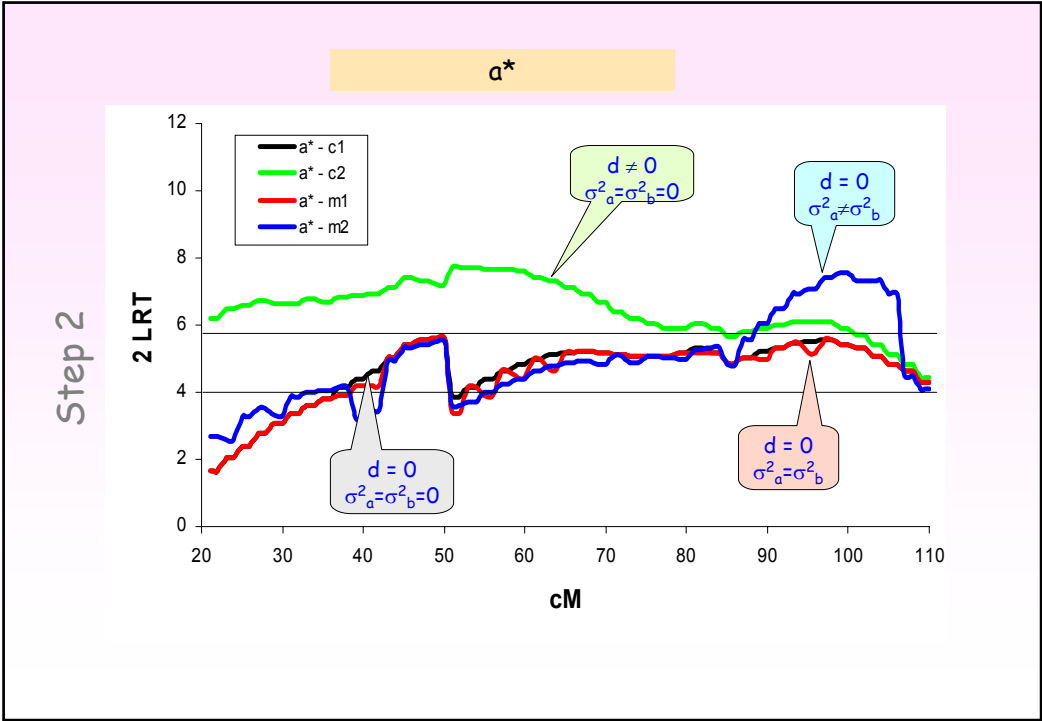
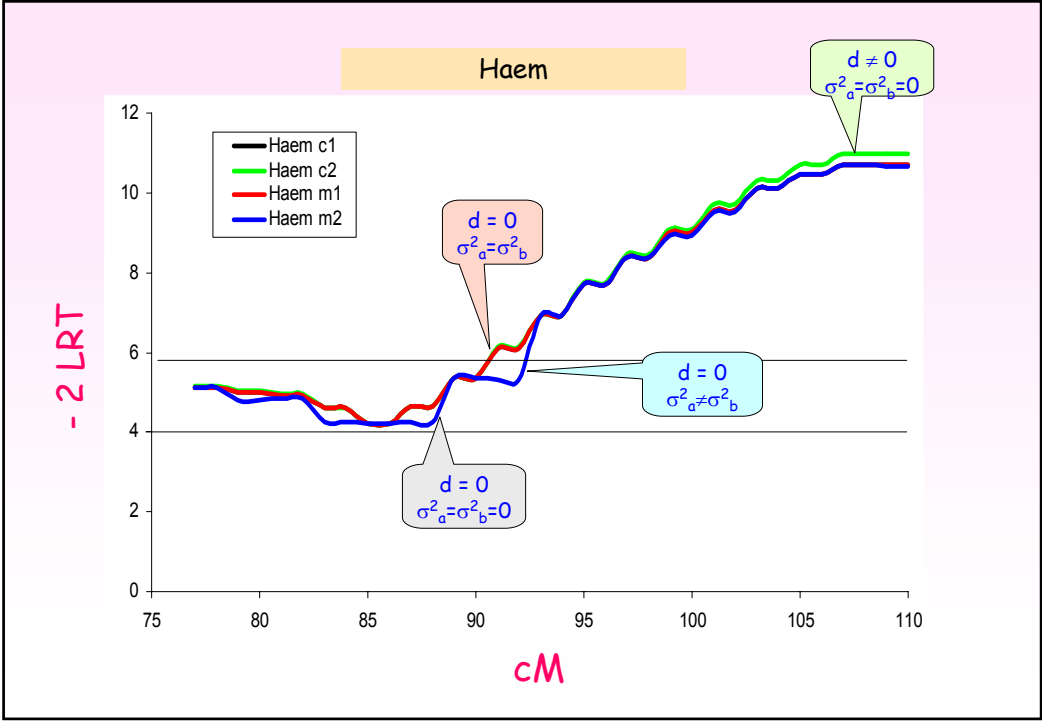
$$\text{Model - m2 : } \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{c}_s \mathbf{a}_s + \mathbf{u}_{sA} + \mathbf{u}_{sB} + \mathbf{u}_0 + \mathbf{e}$$

Step 1

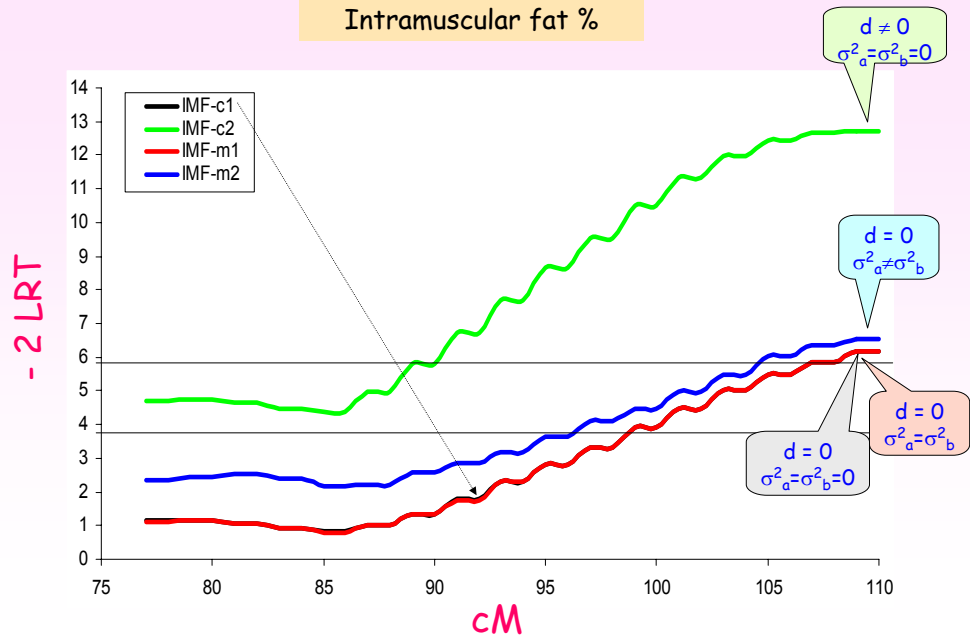


Step 2

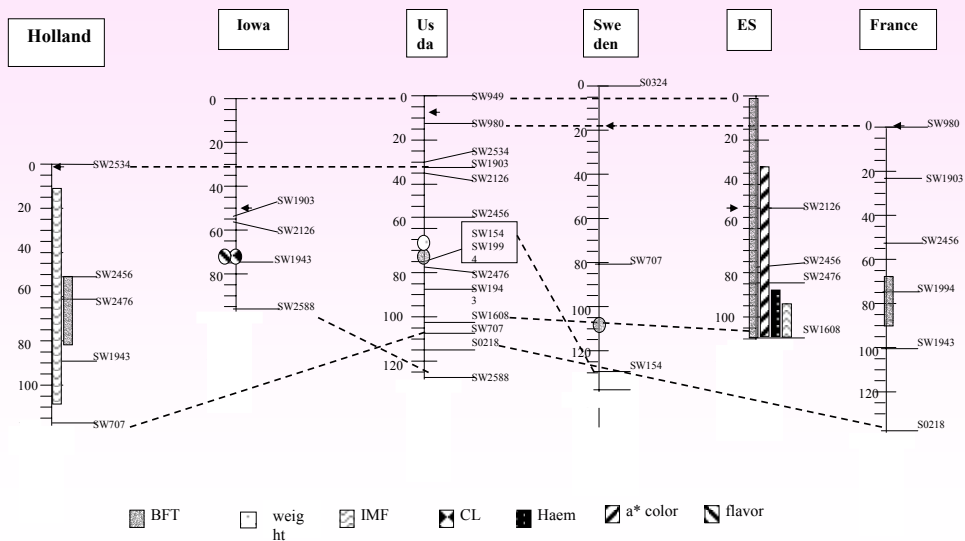




Intramuscular fat %



Comparative X-QTL mapping



Conclusions

A variety of genetic actions are revealed:

Strong overdominance for IMF

Additivity for Haem

Alleles fixed in Iberian, not necessarily so in Landrace

Results for a^* are difficult to interpret (2 QTL?)

Evidence for BFT not conclusive

All QTL experiments coincide in the most promising region

Example 2

Sex chromosome QTL in IBMAP cross

Pérez-Enciso et al. (2002)

Whole genome analysis: wool traits in sheep

Ponz et al. (2001)

Whole genome analysis

Recall...

a genome scan is not the only possible nor feasible strategy, common sense and statistical theory dictates that we should consider jointly all sources of variation.

This seems specially important if we want to discover epistatic relations.

Whole genome analysis: Segment Mapping

(Pérez-Enciso & Varona, 2000)

The **segment mapping** approach consists of dividing the region of interest (e.g., the whole genome) in a series of segments, bounded by arbitrary positions, and trying to obtain the most 'reasonable' partition.

- No distinction between a single QTL or n-QTLs within a segment.
- We are interested in quantifying the contribution to genetic variance of each segment rather than in estimating accurately the position.
- Generalization over classical approaches.
- No 'hierarchies' between segments.

Ponz et al., 2001

Synthetic sheep breed INRA401 = Romanov x Berrichon du cher.

Wool characteristics: staple length, mean fiber diameter, coefficient of variation of fiber diameter.

30 rams, 690 ewes and 1109 phenotyped offspring.

Sparse genotyping, 40 microsatellites distributed in 20 chromosomes out of 26 in the sheep genome.

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \sum_{s=0} \mathbf{g}_s + \mathbf{e}$$

- Which fraction of the genetic variance is explained by typed markers?
- Which is the most reasonable course of action to take?

Analysis steps

1. One initial segment per chromosome; model $M(0, \text{chr})$ vs. $M(0)$ for each chr. in turn.
2. For those significant chrs. ($P < 0.05$), split chrs. in subsegments a, b, c... delimited by each consecutive marker or by half the chr. if only two markers. Choose the combination with maximum likelihood, e.g., max from $M(0,4)$, $M(0, 4a)$, $M(0,4b)$.
3. Assess the fraction of total genetic variance explained by selected segments and assess whether all variance is explained by these segments by comparing model $M(0, i, j, k \dots)$ vs. $M(i, j, k, \dots)$.

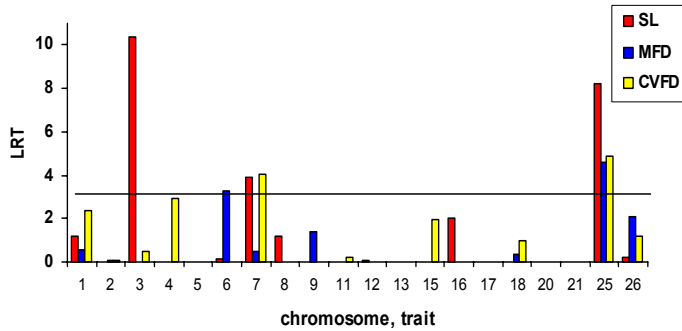


Table 1. Main results of the single chromosome analysis.

Trait	Chr	LRT (<P)	h ₀ ²	h _ε ²
SL	0	—	0.36 ± 0.06	—
	3	10.4 (<5.10 ⁻⁴)	0.16 ± 0.09	0.20 ± 0.06
	7	3.9 (<0.03)	0.21 ± 0.10	0.15 ± 0.07
	25	8.2 (<10 ⁻³)	0.23 ± 0.08	0.13 ± 0.05
MFD	0	—	0.55 ± 0.08	—
	6	3.3 (<0.03)	0.47 ± 0.10	0.11 ± 0.06
	25	4.6 (<0.02)	0.44 ± 0.10	0.11 ± 0.05
CVFD	0	—	0.75 ± 0.07	—
	4	2.9 (<0.05)	0.64 ± 0.10	0.12 ± 0.06
	7	4.0 (<0.02)	0.59 ± 0.10	0.16 ± 0.08
	25	4.9 (<0.02)	0.66 ± 0.09	0.09 ± 0.04

SL, staple length; MFD, mean fiber diameter; CVFD, fiber diameter coefficient of variation; Chr = 0 means that Model (0) was fitted, h₀² thus corresponds to the usual heritability; Model (0, ε) was fitted in all other instances, Chr = ε, where h_ε² should be interpreted as the fraction of additive genetic variance not explained by the particular chr; and h_ε² = σ_ε² / σ_ε²; LRT is twice the likelihood ratio of the Model(0, ε) versus Model(0); approximate probabilities P are obtained from a mixture of χ² distributions, 1/2 χ₀² + 1/2 χ₁².

Single chromosome step

Chromosome dissection step: CV diameter, chr. 4

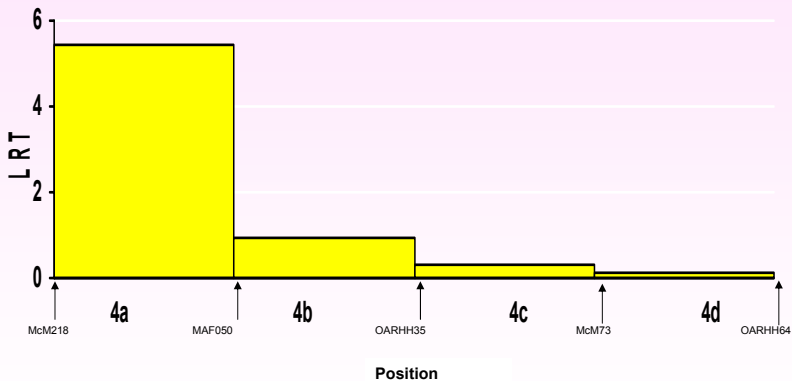


Table 2. Main results of the joint chromosome analysis.

Trait	SL				
	LRT (<P)	h_0^2	h_3^2	h_7^2	h_{25b}^2
a_0 included	0.0 (<0.99)	0.00 ± 0.02	0.16 ± 0.05	0.11 ± 0.05	0.12 ± 0.04
a_0 not included	–	–	0.16 ± 0.05	0.11 ± 0.05	0.12 ± 0.04

MFD			
LRT (<P)	h_0^2	h_6^2	h_{25a}^2
5.2 (<0.01)	0.37 ± 0.10	0.10 ± 0.05	0.11 ± 0.04
–	–	0.23 ± 0.08	0.20 ± 0.07

CVFD				
LRT (<P)	h_0^2	h_{4a}^2	h_7^2	h_{25}^2
3.6 (<0.03)	0.41 ± 0.13	0.15 ± 0.06	0.13 ± 0.07	0.08 ± 0.03
–	–	0.24 ± 0.08	0.31 ± 0.10	0.10 ± 0.04

Global analysis step

We concluded ...

1. All genetic variance for staple length is explained, we should pursue fine mapping.
2. About 60% of total genetic variance is not explained with current typing in mean fiber diameter: pursue genotyping other regions.
3. Results for CV of fiber diameter are intermediate for this trait, about 50% of variance explained: course of action is less obvious.

Day 1. Fine mapping and analysis of complex pedigrees

1. Combining linkage and linkage disequilibrium information
2. Analysis of crosses between outbred lines
3. QxPak software



By M. Pérez-Enciso & I. Misztal

a versatile package for QTL & genetical genomics

Main features

- Multitrait
- Multi QTL
- Different models per trait
- Any number of chromosomes can be analyzed jointly
- Missing observations
- (approximate) Dealing with missing markers
- Flexible QTL modelling
- QTL x other effect (say sex) interaction
- Linkage vs Epistasis tests
- Friendly input file
- Can also be used efficiently for infinitesimal model analyses
- All individuals are included in the analyses

Four grand options

1. Classical REML/ML analyses
2. QTL studies
3. Genetical genomics
4. SNP association studies

Input file

ML_option *
Multitrait_option *
Datafile *
Outfile *
Markerfile *
Haplotypefile *
Number_of_inds
Number_of_qtl
Number_of_effects
Number_of_chromosomes
Marker_positions
Number_of_traits *
Number_of_MCMC_iterations *
Scan_step *
QTL
Effect
Trait
Initial_res_var *
Initial_gen_var *
Test *

QTL modelling

Qtl types defined are:

fix_a: additive fixed effect
fix_d: dominant fixed effect
fix_ad: add+dom fixed effect
snp_a: additive fixed effect (SNP)
snp_d: dominant fixed effect (SNP)
snp_ad: add+dom fixed effect (SNP)
ran_1: additive random effect (common variance to all breeds)
ran_2: additive random effects (different variance per breed)
mix_1a: mixed effect (fix_a + ran_1)
mix_1d: mixed effect (fix_d + ran_1)
mix_1ad: mixed effect (fix_ad + ran_1)
mix_2a: mixed effect (fix_a + ran_2)
mix_2d: mixed effect (fix_d + ran_2)
mix_2ad: mixed effect (fix_ad + ran_2)

Day_1_take_home_message

1. Fine mapping is a risky and very labor intensive task.
2. The main difficulty with complex pedigrees lies in computing IBD probabilities, MCMC methods are the sole means but they are not the panacea.
3. Similarly, Bayesian statistics is very attractive and helpful but does not solve the main problem.
4. Much work remains to be done to combine LD and linkage methods. Assessing the QTL genotype correctly is paramount.
5. A genome scan is not the only possible strategy in a QTL analysis.

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