CHAPTER 1: BREEDING OBJECTIVES

This chapter was compiled by John Gibson from various notes originally produced by John Gibson, The Institute for Genetics and Bioinformatics at UNE, Susan Mezaros, UNE, and Jack Deckers, Iowa State University.

These short course notes are backed by a set of more advanced notes developed by Gibson and Dekkers. The advanced notes are not required reading for the course but are provided for those readers who would like to explore more advanced issues related to the development of breeding objectives.

Breeding objectives, aggregate genotypes and selection indexes

In this short course we will introduce the concepts of breeding objectives, aggregate genotypes and selection indexes. This chapter on breeding objectives deals with the issue of how to set a breeding objective and from that how to construct an aggregate genotype. The next section will deal with how to translate an aggregate genotype into a selection index. The definitions of breeding objective, aggregate genotype and selection index are dealt with in detail below, but briefly can be summarised as:

The breeding objective is the overall goal of the genetic improvement program. This might be to maximise profit, or to maximise economic efficiency or to minimise economic risk.

The aggregate genotype is mathematical function of genetically controlled traits that when maximised will achieve the breeding objective.

The selection index is a mathematical function of traits that are recorded or for which there are genetic evaluations available, that when used for selecting animals for breeding will maximise the aggregate genotype which will achieve the breeding objective.

What is a breeding objective?

The **breeding objective** is the overall goal of our breeding program. By setting a clear breeding objective it is possible to then make objective decisions in breeding programs, such as:

1) Choice of animals as parents in within-line or within-breed selection
2) Choice of which lines or breeds to introduce to the production system
3) Evaluation of different investments in breeding programs and design of alternative breeding programs. The breeding objective provides the criterion to quantify and then maximize the return on investments in the breeding program.

A breeding objective need not be economic. For example, in many companion animal species it is tempting to believe that the breeding objective must be maintenance of ridiculous appearance and congenital abnormalities. But in the context of this course we will assume that the breeding objective is essentially an economic objective.

An obvious and attractive economic breeding objective would be to maximize profit, and, for many years, it was essentially taken as read by most animal breeders that this was the breeding objective. This acceptance was, however, more because little attention was given to defining breeding objectives rather than because of any great conviction on the part of animal breeders.

As will be discussed, maximizing profit still appears to be a logical goal for most, perhaps all, animal breeding programs. But there are a few tricky issues still to be resolved, such as whose profit is being maximized? Is this profit to the producer (farmer), or the breeding organization or the processor or the retailer or the consumer or the whole industry? A breeding company, for instance, might see clearly that it is their own profit they wish to maximize. But do they do that by maximizing the producer's profit or the processors' profit or someone else's profit or some combination of other people's profit? Or indeed, is it related at all to other people's profit?

Economics

Economics has become synonymous with money, but in fact is defined as the study the balance of inputs and outputs of a system. When examining the economics of a given system it is useful to express all inputs and outputs on a common scale. For many human enterprises, the common scale for measuring inputs and outputs is very often monetary; leading to the common misconception that economics is always monetary. In this Chapter the examples are indeed expressed in monetary terms, and this seems reasonable given that the majority of players in farming will only remain in business in the long terms if they remain profitable. But there are aspects of farming enterprises that can be difficult to express in monetary terms and so are often left out of the definition of breeding objectives. An example is the personal pleasure a farmer may gain from the quality of the appearance of their animals or the landscape of their land, which may lead farmers not to maximise the monetary returns from their farming enterprise. In such a situation, the farmer may still be maximising his/her profit, but their definition of profit includes returns, such as the pleasure they obtain from the appearance of their animals or land, that are not monetary. There are methods that can convert such non monetary returns into monetary equivalents by assessing how much monetary profit someone is willing to forego to achieve a non-monetary return. But such methods can difficult to apply in practice. When defining breeding objectives it is often simpler and nearly as effective to allow that certain aspects of a farm production system are not captured by the monetary assessment of the enterprise and to leave room in the final decisions on selection of animals inclusion of traits that are not captured in the breeding objective defined in terms of money. The danger in this approach is that too much emphasis is placed on traits that do not contribute to the monetary success of the business. But there are simple methods to measure how much monetary improvement is lost when allowing for other traits in the selection decisions and the selection decisions can be modified to achieve a balance between monetary and other genetic gains.

In general, who obtains extra profit? Consider the farmer who uses animals genetically improved for growth to slaughter, be they beef cattle, pigs, poultry, fish or whatever. These animals will grow faster and more efficiently therefore costing less and, if quality is improved, perhaps bringing in more income. The farmer’s profit increases. If they were the only farm with genetically improved animals they could maintain that increased profit. But simple market economics tells us that if many farmers increase their profits, this translates to a more efficient production system which, with competition, leads to a reduction in prices. Thus some, perhaps all of the farmer's increase in profit is eventually lost to the processor, the retailer and eventually the consumer.

Gordon Dickerson, one of the founders of modern animal breeding theory and practice, recognized some of these problems and concluded that in a competitive world the only reasonable breeding objective was economic efficiency, defined as the ratio of production income divided by production costs. Taking an industry-wide perspective, particularly from the consumer's point of view, economic efficiency has a certain appeal. It is a measure that maximizes the difference between value and cost and is independent of the size of the production system. But it still faces the problem that a breeding organization and their clients, the producers, will both prefer to maximize their net income and will be little concerned with efficiency unless maximum efficiency equates with maximum profit. See the more advanced notes for a more detailed discussion.

Throughout most of this Chapter we will assume that the breeding objective is to maximize the profit of the farm enterprise. In this context we go on to quantify the effects of genetic change on profit, along with the consequences for deriving economic weights for use in multiple trait selection. Some of the differences between different perspectives will be mentioned. The material provided here necessarily covers only a few basic principles.

Dealing with multiple traits

The aggregate genotype

In all cases of genetic improvement more than one trait is to be improved. So, having defined the breeding objective, it becomes necessary to define the relative importance of the traits to be improved that will contribute to the overall breeding objective. This involves first identifying which traits might be genetically improved and then determining the economic value (referred to as the **economic weight**) of improving each of those traits. For a given animal that is a candidate for selection, the sum of its additive genetic values multiplied by the economic weight for each trait is referred to as the **aggregate genotype**,. i.e.

$$H = v_1 g_1 + v_2 g_2 + v_3 g_3 \ldots\ldots \text{etc}$$

Where H is the aggregate (economic) genotype, v_1, v_2 etc are economic weights of traits 1, 2, etc, and g_1, g_2, etc are the additive genetic values of traits 1, 2, etc. for however many traits are included.

The selection index

In practice, the additive genetic value of the various traits for each individual are not known. However we can record each individual's performance for a number of traits. The observations on these traits can then be combined into a **selection index**, *I* of the form,

$$I = b_1 x_1 + b_2 x_2 \ldots b_m x_m$$

where x_i is an observation on the i^{th} trait and b_i is the selection index coefficient (or weight) for that trait. The same principle applies when instead of having performance data on each individual when a set of estimated breeding values (EBV) coming from a genetic evaluation program.

The problem is then to estimate the selection index coefficients, b_i, such that selection of individuals on their **selection index** value, I, maximizes response in the **aggregate genotype**, H. Part 2 of this short course covers the methods involved in estimating selection index weights and applying selection indexes in practice. We here return to the question to estimating economic weights to obtain an aggregate genotype and how that relates to the breeding objective.

Choice of traits to include in the aggregate genotype versus selection index

The purpose of the aggregate genotype, i.e. describing genetic variation of the breeding objective in terms of biological traits, determines the criteria for deciding which traits to include in the aggregate traits:

- In principle, all traits that directly contribute to the breeding objective should be included
- Traits that have an indirect impact on the objective (e.g. indicator traits) do not belong in the aggregate genotype (they belong in the index)
- Traits that have little or no genetic variation do not need to be included (note that low heritability does not necessarily imply low genetic variation).

In contrast, criteria for inclusion of traits in the selection index are:

- The trait must be recorded such that EBV can be obtained on selection candidates
- The trait must have reasonable heritability, (although low heritable traits can provide accurate EBV if sufficient data is available, such as a progeny test).
- The trait must be one of the traits in the aggregate genotype or be genetically correlated to one or more traits in the aggregate genotype.

In development of breeding goals and selection indexes, a clear distinction must be made between economic traits that are included in the breeding goal and indicator traits that are included in the selection index. With regard to interpretation of the selection index, this involves clarification of the role of indicator traits in relation to the economic traits in the breeding goal. For example, a frequent assertion of breeders is the need to include conformation traits in the breeding goal. Although conformation traits can have a direct economic value for breeders who sell breeding stock, conformation only has an indirect economic value in a commercial milk production environment through its relationship with herd life and functionality. In this case, conformation traits should not be in the breeding goal but belong in the selection index as indicator traits for components of the breeding goal. Note that recording a trait is not a requirement for including a trait in the aggregate genotype but it is for traits to include in the index.

In development of the aggregate genotype many alternative traits and trait definitions can be considered for inclusion. For dairy cattle, traits can be generally classified as milk production traits (milk, fat, and protein), reproductive performance traits, health traits, and feed efficiency traits. Workability traits (e.g., temperament and milking speed) are included in some instances also. In a review, Groen et al. (1997) included milk production, days open, clinical mastitis, milking labour, ketosis, milk fever, displaced abomasum, and laminitis as traits in the aggregate genotype. An aggregate genotype that consists of production traits and herd life is frequently used as a simplified breeding goal (Dekkers and Jairath, 1994). In such a breeding goal, traits associated with health, reproduction, and workability are compounded into the trait herd life. The advantages of such a breeding goal are that fewer economic and genetic parameters need to be estimated and that it is easier to explain to producers. Using herd life instead of individual traits does, however, reduce the completeness of the breeding goal; Allaire and Keller (1993) estimated that a breeding goal of production and herd life would leave 15% of genetic variation in economic merit unaccounted for. The impact on efficiency of the resulting selection index, however, will be less if phenotypic data on health and fertility traits are unavailable. A logical extension of a breeding goal based on production and herd life is to include udder health, as has been proposed in several studies [e.g., (Colleau and Le Bihan-Duval 1995, Dekkers 1995)].

In some cases, proper choice of traits to include in the aggregate genotype can lead to significant simplifications, for example in relation to genotype by environment interaction (GxE) or non-additive genetic effects. Bourdon (1998) suggested the use of physiological traits for inclusion in the breeding goal to avoid such complications. For example (Goddard, 1998), slaughter weight of beef cattle in the tropics is an economically important trait and could, therefore, be included in the breeding goal. However, this trait has the potential for high levels of GxE. Traits such as growth potential and adaptation to tropical environments are physiological traits that are expected to be less affected by the environment and would be good predictors of slaughter weight under a range of environments. Thus, their inclusion would make the aggregate genotype more generally applicable to a wider range of environments.

Care must be taken, however, not to leave traits out of the aggregate genotype that could lead to suboptimal decisions. For example, ignoring fertility and health could lead to overestimating the benefits associated with increasing yield.

Methods for estimating economic weights

Following the definition of the aggregate genotype, the economic value of trait i, v_i, is defined as the effect of a marginal (one unit) change in the genetic level of trait i (g_i) on the breeding objective, keeping all other traits that are included in the aggregate genotype constant. On the basis of this definition, three general methods for derivation of economic values have been used:

1) **Accounting method**: in this method, the economic value is derived as returns minus costs:

$$v_i = r_i - c_i$$

Where r_i is the extra return received from a one unit increase in the mean for trait *i*, and c_i is the extra cost associated with a one unit increase in the mean for trait *i*. For example, considering milk yield for dairy cattle, r_i is the return per kg increase in milk yield, and c_i is the extra feed cost associated with a one kg increase in milk yield.

In this accounting procedure, it is important to avoid double counting. For example, when fat and protein yield are also included in the aggregate genotype, extra returns and costs associated with a one kg increase in milk yield must be computed while keeping the means for fat and protein yield constant. Even though in practice an increase in milk yield tends to be associated with increases in fat and protein yield because of positive correlations between these traits. Not doing so would result in double counting because the economic effect of increasing fat and protein **yield** are also accounted for in the economic values of these respective traits.
In addition, it is important to realize that r_i and c_i are marginal rather than average returns and costs. Thus, they must be evaluated on the basis of a marginal increase of the trait value above its current value.

2) **Profit function**: in general, a profit function is a single equation that describes the change in net economic returns as a function of a series of physical, biological and economic parameters. As will be shown in section 7.3, the economic value of trait *i* can be obtained as the first partial derivative of the profit function evaluated at the current population mean for all traits. The profit function method avoids double counting because of the use of partial derivatives. In addition, because of their mathematical properties, profit functions facilitate theoretical derivations of economic values and have been used extensively for that purpose, as will be demonstrated in the following sections.

3) **Bio-economic model**: production systems are complex and can often not be described by a single profit function. In a bio-economic model, relevant biological and economic aspects of the production system are described as a system of equations. Examples of bio-economic models are in the Tess et al. (1983, J. Anim. Sci. 56:354) for pigs and Van Arendonk (1985 Agric. Systems 16:157) for dairy cattle. Both these models describe the life cycle of an animal, including inputs and outputs, as a function of biological traits and economic parameters.
Bio-economic models can be used to derive the economic value of trait *i* in the following manner:
   - *1°* Run the model for current population means for all traits, including the current mean for trait *i*, m_i, and record the average profit per animal: P_{m_i}
   - *2°* Increase the mean of trait *i* increased by D to m_i+D, while keeping the means of other traits at their current values, run the model and record the average profit per animal: P_{m_i+D}
   - *3°* Derive the economic value for trait *i* as: $v_i = \dfrac{P_{m_i+D} - P_{m_i}}{D}$

A simple production model

As noted above, we here assume that the overall breeding objective is to maximise farm profit. A simple production model takes into account the income and expenses that are related to the traits that might be improved. In our simple example of a sheep enterprise we have 3 genetically controlled traits that contribute to profit (i.e that contribute to the aggregate genotype). We could (and in practice usually should) have included many other traits, but for this example we will restrict it to 3 genetically controlled traits.

1. Weaning rate (**nl**); this is the number of lambs that each ewe, on average, weans. Currently this trait has an average of 1.2 lambs weaned per ewe per year.

2. Days to reach slaughter (**d**); this trait describes the average time taken for lambs to reach slaughter weight after weaning. The average time for our lambs is 100 days.

3. Greasy fleece weight (**w**); this is the amount of wool each ewe, on average, shears. Our ewes shear on average 3.5 kilograms wool.

We can now build up our production model by including other constant values that in our model are either not under genetic control, or which we are not interested in changing through genetic improvement. In our simple model these traits are:

- Market or sale weight (**sw** = 40 kg)

We also need to include the prices for product sold and costs incurred:

- Sale price per kilogram lamb, $\mathbf{p_l}$ = $1.00/kg
- Net wool price per kilogram, $\mathbf{p_w}$ = $3.00/kg
- Cost per lamb per day to keep, $\mathbf{c_l}$ = $0.25/day
- Cost per ewe per year to keep, $\mathbf{c_e}$ = $10.00/year

Note that to keep this example simple we have not included a cost for shearing and delivering the wool to sale, so the net wool price is assumed here to be the net value of the wool after the farm has factored in its costs of collecting and delivering the wool to sale.

We can now include all of this information in a small income and expenses statement and calculate the profit of the enterprise:

Traits	**Averages**
Weaning rate (nl)	1.2
Days to Slaughter (d)	100
Wool weight (w)	3.5

Constants	
Weaning weight (ww)	20
Sale weight (sw)	40

Prices	Costs	Income
Lamb price per kg (p_l)		\$1.00
Lamb Cost per day (c_l)	\$0.25	
Wool price per kg (p_w)		\$3.00
Annual cost per ewe (c_e)	\$10.00	

Calculation of net income		
Wool income per ewe	(3.00\$/kg X 3.50 kg)	\$10.50
Net income per lamb	(40kg X 1.00\$/kg - 0.25\$/day X 100 days)	\$15.00
Net income per ewe	(\$10.50-\$10.00 + 15.00\$/lamb X 1.2 lambs)	\$18.50
Net income per flock of	10,000 ewes	\$185,000.00

Profit Function

We could have written the production model in a profit function form as:

Profit = number of ewes X (net wool income per ewe + net income per lamb X number of lambs per ewe)

$$P = ne \text{ X } ((w \text{ X } p_w - c_e) + (sw \text{ X } p_l - d \text{ X } c_l) \text{ X } nl)$$

We have now expressed the breeding objective in three different ways; as an informal description of the traits we want to improve and in what direction, as a formal production model, and as a profit function. Now we can proceed to determine the relative importance of the traits by deriving the **economic weights** of the traits.

Derivation of Economic Weights

The **economic weight** of a given trait is defined as the rate of change in profit as that trait is improved when holding all other traits unchanged. We can estimate the economic weight of each trait directly from the production model, or by calculation partial first derivatives of our profit function. When genetic change is small, both approaches result in the same value.

1. Economic weights from the production model.

If we increase one of the traits by a small amount and look at the change in income, we obtain the economic value of that trait. For example, improving **nl**:

Traits	Averages
Weaning rate (nl)	**1.3**
Days to Slaughter (d)	100
Wool weight (w)	3.5

Constants	
Weaning weight (ww)	20
Sale weight (sw)	40

Prices	**Costs**	**Income**
Lamb price per kg (p_l)		$1.00
Lamb Cost per day (c_l)	$0.25	
Wool price per kg (p_w)		$3.00
Annual cost per ewe (c_e)	$10.00	

Calculation of net income		
Wool income per ewe	(3.00$/kg X 3.50 kg)	$10.50
Net income per lamb	(40kg X 1.00$/kg - 0.25$/day X 100 days)	$15.00
Net income per ewe	($10.50-$10.00 + 15.00$/lamb X 1.3 lambs)	$20.00
Net income per flock of	10000 ewes	$200,000
Base net income (nl=1.2)		$185,000.00
Profit due to 0.1 extra lamb per ewe (nl=1.3)		**$15,000**

Since we increased weaning rate by 0.1 lambs per ewe, the economic value per unit increase in the traits (i.e increase by 1 lamb per ewe) is $15,000/0.1 = 150,000 $ per lamb per flock, or $150,000/10,000 = 1.5 $ per lamb weaned per ewe.

We can follow the same approach for each of the other traits to derive their economic weights. Try increasing the days to slaughter by one and then the wool weight by 0.1 kg. You should obtain economic weights of -3,000 $ per increased day to slaughter per flock and 35,000 $ per kg wool per ewe per flock; or -0.3 $ per increased day to slaughter per ewe and 3.5 $ per kg wool per ewe per ewe. Note that the negative sign for economic weight for days to slaughter indicates that increasing days to slaughter decreases profit. Thus the negative sign indicates that the desirable direction of genetic improvement is to decrease days to slaughter.

2. Deriving economic weights using partial first derivatives of the profit function.

Firstly, a quick refresher on calculus. (see appendix 1 for more detailed refresher). Recall that the partial first derivative of a function $\boldsymbol{y}$, with respect to the variable $\boldsymbol{x}$, is $\delta \boldsymbol{y}/\delta \boldsymbol{x}$. It is simply the slope of the line at the point $(\boldsymbol{x,y})$.

- If the line is a straight line, it is the slope of the line.
  If $\boldsymbol{y = m\,x + b}$, then $\delta \boldsymbol{y}/\delta \boldsymbol{x} = \boldsymbol{m}$.
- If the line is a curve, it is the tangent at the point (x,y). eg for a quadratic curve:
  If $\boldsymbol{y = m\,x^2 + b}$, then $\delta \boldsymbol{y}/\delta \boldsymbol{x} = \boldsymbol{2\,m\,x}$.

Notice that the constant ***b*** doesn't appear in the partial first derivative. Only the coefficient and exponent of the variable for which the derivative is calculated appears in the partial first derivative.

The key point to note is that for non-linear functions, the partial first derivative (i.e. the economic weight) depends on the current flock average for that trait.

- If $\boldsymbol{y = m/x + b}$, then $\boldsymbol{y = mx^{-1} + b}$, then $\delta \boldsymbol{y}/\delta \boldsymbol{x} = \boldsymbol{- m\, x^{-2}}$

To calculate the first derivative of the profit function with respect to greasy fleece weight, $\boldsymbol{\delta P/\delta w}$, write the profit function:

$$P = ne \times (w \times p_w - c_e) + ne \times (sw \times p_l - d \times c_l) \times nl$$

$$\begin{aligned} v_w = \delta P/\delta w &= ne \times p_w \times w^0 \\ &= ne \times p_w \\ &= 10{,}000 \times \$3.00/\text{kg} \\ &= \$30{,}000/\text{kg} \end{aligned}$$

Applying the same approach to days to slaughter, d:

$$\begin{aligned} v_d = \delta P/\delta d &= -ne \times d^0 \times c_l \times nl \\ &= -(10{,}000 \times \$0.25/\text{day}) \times nl \\ &= -\$2{,}500/\text{day} \times nl \\ &= -\$3{,}000/\text{day} \end{aligned}$$

In this case, notice that our first derivative now includes one of the other traits, nl. We know the flock average number of lambs per ewe, so we can substitute this in for nl to get the economic value of d. What this is telling us is that the economic value of days to slaughter depends upon the average number of lambs weaned.

We see the same for nl:

$$\begin{aligned} v_{nl} = \delta P/\delta nl &= ne \times (nl^0 \times sw \times p_l - nl^0 \times d \times c_l) \\ &= ne \times (sw \times p_l - d \times c_l) \\ &= 10{,}000 \times (40\text{ kg} \times \$1/\text{kg} - d \times \$0.25/\text{day}) \\ &= \$400{,}000 - 10{,}000 \times d \times \$0.25/\text{day} \\ &= \$400{,}000 - \$250{,}000 \\ &= \$150{,}000/\text{lamb} \end{aligned}$$

Notice that these partial first derivatives are equal to the economic weights we obtained with the production model! We have calculated these economic weights on a *per flock* basis. This is the same as if we had calculated it on a *per ewe* basis because we can divide all of our economic weights by the number of ewes in the flock. For example, the economic weight for greasy fleece weight (w) is $3.00 per ewe, which is the net price the producer receives for 1 kg extra sold (in this model the cost of collecting and marketing the wool is included in the net wool price).

3. Absolute versus relative economic weights

The economic weights estimated above are calculate in absolute terms, for example as $ per lamb born, or $ per day to market. There is a 50 fold range in the economic weights between days to market at $3000/day and number of lambs born at $150,000 per lamb. But this gives a misleading picture of the value of genetic improvement of the different traits because it takes no account of how easy it is to change days to market by one day versus changing number of lambs weaned by one lamb. Obviously in this case it is much easier to genetically improve sheep to reduce days to market by one day than to genetically improve number of lambs weaned by a whole extra lamb.

The rate of change it is possible to make in different traits depends on many factors, including the design of the genetic improvement program, which traits are recorded, which traits have EBV, how accurate are the EBV and so on. We will see in the next chapter how economic weights are used in a breeding program and how that ultimately results in genetic improvement of overall profit and improvement of individual traits. But it is often very instructive to get a rough idea of the relative value of genetic change in different traits at the time that one estimates the economic weights.

As seen earlier in this course, response to single trait selection is given by,

$$R = ih^2\sigma_p$$

Since intensity of selection, i, is determined by the breeder, response is proportional to $h^2\sigma_p$, and $h^2\sigma_p$ can be defined as a unit of response (i.e the response expected from phenotypic selection within an intensity, i, of one unit). Thus if we multiply each absolute economic weight by the response unit for that trait we can obtain a relative economic weight which is an estimate of expected economic response to selection on that traits.

In the current example, the phenotypic s.d., heritabilities, response units and relative economic weights are shown in the following table.

	nl	**d**	**w**
Absolute economic weight	$150,000.00/lamb	-$3,000.00/day	$30,000.00/kg
Phenotypic s.d. (σ_p)	0.60	13.0	0.60
Heritability (h^2)	0.1	0.3	0.3
Response unit ($h^2\sigma_p$)	0.06 lambs	3.9 days	$0.18 kg
Relative economic weight	$9,000	-$11,700	$5,400

The results in this case indicate that the greatest potential for economic response is for days to market, with number of lambs weaned having slightly less potential and wool weight having the least potential.

4. More complicated models

The example we worked with above is highly simplistic in a number of ways. Firstly, the relationship between income and costs and genetic change is generally more complicated than shown above. For example, we assume a fixed cost per day to market. In practice, a faster growing lamb will have a higher cost per day to maintain and grow, because it is growing faster. As another example, there are no costs associated with increasing weaning rate, whereas in practice as average litter size goes up ewes will require more feed for pregnancy and lactation and survival of ewes and lambs will likely decrease as average number of lambs born increases. These relationships will likely also be non-linear, with lamb and ewe survival decreasing more rapidly as litter size increases. An introduction to dealing with the more complicated models required to approach a reasonable model of reality is provided in the more advanced notes.

5. Dealing with non-linear models

In most realistic situations P is not likely to be a linear function of performance traits. In general, P might be any more or less complex function of performance traits,

$$P = f(y_1, y_2 \ldots y_n)$$

and the general solution to v_i with a linear aggregate genotype is the partial derivative of profit with respect to y_i evaluated at the current mean for all traits:

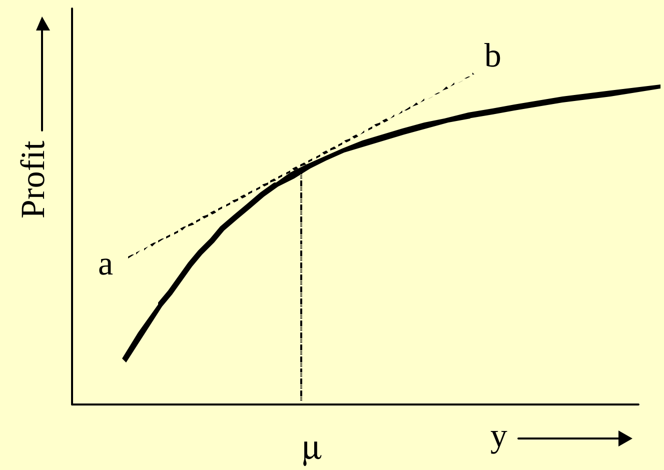
$$v_i = \frac{\partial f}{\partial g_i}[\mathrm{m}]$$

$\frac{\partial f}{\partial g_i}[\mathrm{m}]$ is the partial derivative of the profit function with respect to g_i evaluated at the current mean, m. This partial derivative is the rate of change in profit as genetically controlled performance of trait i changes, when all other traits remain unchanged. In other words it is the (linear) tangent to the profit curve with respect to y_i, at the mean performance of all other traits (see Figure 7.1). Substituting these economic values in the aggregate genotype results in:

$$H = \frac{\partial f}{\partial g_1}[\mathrm{m}]\, g_1 + \frac{\partial f}{\partial g_2}[\mathrm{m}]\, g_2 + \ldots + \frac{\partial f}{\partial g_n}[\mathrm{m}]\, g_n$$

This shows that the aggregate genotype is a first order Taylor series approximation of the profit function evaluated at current population means.

Figure 7.1: Generalized curvilinear relationship between trait yield, y, and profit, showing the tangent, ab, to the profit curve at the population mean, μ.



Use of first partial derivatives evaluated at the current population mean requires that genetic change is sufficiently small so that second order effects can be ignored. Illustrative examples are given below. Exact solutions, not requiring the genetic change to be very small, are given later.

The situation is illustrated graphically in Figure 7.1, which shows a curvilinear profit function of a single trait, *y*, where the rate of increase of profit decreases as the trait mean increases. The economic weight of *y* is the slope of the tangent to the profit curve at the population mean, m, shown by the straight line, from a to b. It is clear from this figure that using this tangent to the profit curve in a linear prediction of aggregate genotype should be a reasonable representation of the true curve, if the range of genotypes being considered is small relative to the curvature of the graph.

In most cases, the profit function is described in terms of population means:

$$P = f(m_1, m_2, \ldots , m_n) = f(m)$$

and economic values are derived as partial derivatives with regard to the population mean:

$$v_i = \frac{\partial f}{\partial m_i}[m]$$

A specific example of a non-linear profit function is given in Appendix B. The example illustrates how in real life deriving economic weights as partial derivatives of a profit function would do a poor job when comparing genotypes with large differences in performance, such as when comparing two different breeds. But these same economic weights do a good job when the range of genotypes is relatively small, such as when comparing animals as candidates for breeding within a population.

6. Perspective from which profit is viewed

Great care has to be taken to be clear on the appropriate level in the production system from which one estimates change in profit due to selection. In the sheep example above, different absolute and relative economic weights are obtained if profit is calculated per lamb marketed rather than per ewe (try reworking the example for a per lamb basis and see for yourself). We have already seen above that in the sheep example the relative economic weights are the same whether expressed per ewe or as change in profit of the whole farm. This results from the assumption that genetic improvement does not affect the number of ewes on the farm. In many cases this assumption may not be realistic and if the number of ewes has to change because of genetic improvement, then economic weights expressed per ewe will not equal those expressed per farm. As far as a producer is concerned it will usually be most appropriate to estimate profit of the whole production unit. The production unit will often be the farm, but it could also be the whole production industry, because many producers, often the whole industry are participating in genetic improvement and the impacts of genetic improvement cannot of one farm cannot be isolated from improvement on other farms. For example, in our sheep example it is realistic to expect that if a single farmer improves the number of lambs produced that they will be able to sell those lambs. But if the whole industry improves lamb numbers the increased supply of lambs may drive lamb prices down. Allowing for the expected drop in lamb prices would then lead to a higher relative economic weight being put on traits that reduce costs of production (such as days to market) versus traits that increase output. For more detailed discussion on perspectives in genetic improvement, see the more advanced notes. The subject is also revisited in section 11 (rescaling in genetic improvement), below.

7. Future versus current management systems

One school of thought would say that the appropriate management system is that which will be (or is most likely to be) in place when genetic improvement is utilized. This recognizes that there is considerable time lag between a selection decision being taken and when improved animals resulting from that decision enter the production system. For example, with swine in a breeding company, a selected boar will have progeny in the nucleus next year which will pass genes through one or two levels of multiplier herds over the next year or two and will finally result in genetic improvement in a commercial herd anything from 2 to 4 years hence. A farmer choosing an AI bull for use in his dairy herd today, will see replacement heifers starting their first lactation about 3 years from now, and they will hopefully stay in the herd for 4 or 5 years. So his decision today results in improved profitability over the period 3 to 8 years from now. On the other hand, the sire-selector, setting up matings to produce a potential young bull for progeny testing should be looking over 10 to 15 years ahead (see Table 3.1).

Table 7.1 Approximate time scales of genetic improvement in dairy cattle.

Event	Time from previous event (yr)	Cumulate time (yr)
Mating to produce young bulls	0	0
Young bull born	0.75	0.75
Youg bull 1st service to produce daughters for progeny test	1.25	2.0
Progeny test daughters born	0.75	2.75
Progeny test daughters complete 1st lactation; 1st proof on bull available	3.0	5.75
Average time for widespread use of proven sire; 2 years of use	1.0	6.75
Main crop daughters born	0.75	7.5
Main crop daughters start 1st lactation	2.25	9.75
Main crop daughters complete average lactation (assume average herd life of 3 yr)	3.0	12.75

Imagine a profit equation for dairy production, which showed that the relationship between profit and genetic improvement for milk production per cow is dependent on the initial production level of the herd (a simple example is given in section 3.3). If the current rate of increase in yield per cow is 2% per annum, due to both genetic and management improvement, then the expected production level 13 years from now will be proportionately $(1.02)^{13} = 1.29$ (i.e. +29%) higher than today. From Table 3.1, the sire selectors decision results in genetic improvement at an average of about 13 years from now, and so his profit equation, used to derive economic weights, should be evaluated at an average herd production level 29% higher than the present level. If the economic value of yield is a linear function of herd level of production, this has no impact. But in most cases the economic values of one or more traits are non-linear functions of the herd level of production.

8. Optimum management systems

A variant on the perspective of profit equations for future management systems is profit equations for optimum management systems. The argument is that genetic improvement is a slow but cumulative process and consequently works most effectively when the direction of change is consistent over long periods. It is important therefore not to breed for sub-optimum management systems, since non-genetic improvement in management are generally made more rapidly and easily than genetic improvements. We should therefore breed only for those aspects of improvement that cannot easily be made by other management improvements.

9. Future technologies

In some cases impending technologies can radically affect profit equations by eradicating or perhaps creating opportunities for genetic improvement. A good example might be disease resistance. Imagine a particular viral disease of swine, say, which is estimated to cost an average of $4.00 per slaughter pig in terms of prophylactic treatments and reduced performance. Genetic resistance to this disease could appear in the profit equation and might potentially be an important component of profit. However, if an effective vaccine is discovered which costs, say, $0.2 per pig to administer and completely prevents the disease, then the potential value of complete genetic disease resistance falls from $4.00 per pig to $0.2 per pig, the value of not having to vaccinate pigs. In this situation a great deal of effort and expense might be wasted in selecting for resistance if in the interim an effective and cheap vaccine is discovered.

10. Future markets

The same arguments applied to future management and technologies apply to markets. Being a long-term cumulative process, genetic improvement should anticipate future market prices. A good example was the US pig market where pig producers were not paid premiums for lean carcasses for many years after consumer preferences had shifted from fat to lean pork. It was obvious that at some point farmers would have to be paid incentives to produce lean carcasses, and several breeding companies anticipated this change, preparing lines of pigs ready to meet the demand for lean carcasses. Breeding companies that failed to anticipate this change lost substantial market share when the change came.

An example of almost universal failure to anticipate market changes, was the North American and European dairy cattle breeding programs that tended to have their selection indexes lag well behind changes in milk pricing. Driven by changing consumer demand and market oversupply of milk components, pricing to farmers shifted progressively from payment on milk volume, through payments on fat and protein through to heavy emphasis on protein. Given the time lags in dairy cattle genetic improvement, the selection indexes should ideally have led changes in milk payments by 10 to 15 years. The damage caused by failure to anticipate market changes in this case was not as high as it might have been because of the fairly high genetic correlations between milk volume, milk fat and milk protein yield meant that radical changes in economic weights had a smaller effect of selection indexes and directions of genetic change. But even so, many years of potential genetic gain were lost.

11. Rescaling of genetic improvement

The issue of rescaling has been touched upon at several points above. Rescaling applies to the fact that many forms of genetic improvement will lead to changes in outputs or inputs that will require the farm enterprise to be rescaled. An obvious example was provided in Appendix B, where genetic increase in milk yield forces the farmer to reduce the number of cows because he/she has to stay within their quota for milk production[1]. In this case, the farmer has to **rescale** the enterprise by reducing the number of cows to keep

[1] Exactly the same argument applies when considering genetic improvement across the whole industry, because there will be a fixed quota for total milk supply in that industry. The phenomenon is seen very dramatically in countries in which virtually all milk is sold on the internal market which already has an oversupply of milk, such as USA, Canada, EU. Cow numbers have dropped dramatically as production levels have gone up (due to both genetic improvement and better management), while total milk production has remained relatively constant over the past 30 years or so.

output constant after genetic improvement. The economic weights are estimated after allowing for this rescaling of the enterprise.

In our sheep example, genetic improvement will increase the outputs (number of lambs and amount of wool sold) of the farm. But if the extra feed is available to support the higher production level, then we could increase the output immediately by increasing the number of ewes stocked on the farm, without waiting for results of genetic improvement. So the real value of genetic improvement should be the difference between what we can get via genetic improvement versus what we can achieve by alternative changes, such as increasing stocking density. In this case, when estimating economic weights we should **rescale** the enterprise so that outputs from genetic improvement match those from the best alternative management change (increased stocking density in this case).

Alternatively, it might be that stocking density is already at a maximum in our sheep example; eg because current stocking density uses up all available pasture supply and the price of purchased feed makes buying in feed not cost effective. In that case, a genetic increase in number of lambs born and wool produced will require extra feed resources that are not available. Thus we will need to **rescale** the enterprise, by reducing stocking density after genetic improvement, so that total feed consumed remains constant.

The three examples above are for rescaling to constant output (in case of milk quota), rescaling to match alternative management changes (in case of increased stocking rate) and rescaling to constant inputs (in the case where stocking density is already at a maximum on the farm). In all cases, rescaling is required because there is some form of constraint on the production system that needs to take into account when estimating the economic value of making genetic improvement.

Methods of dealing with such situations are dealt with in detail in the advanced notes. There are methods that involve algebraic modifications to unconstrained profit equations to allow for any rescaling that is required. Alternatively, a bioeconomic model can be constructed that models how the production system responds to changes in management or genetics, allowing for any number of constraints that operate on the production system. In many ways the latter approach is probably the safest, as it forces the researcher to identify and model all constraints explicitly and it is easy to see when the model is failing to deal adequately with constraints (because it will produce unrealistic results in terms of inputs and outputs after genetic or management change).

11. Developed world vs developing world considerations

The principles outlined here apply to any production system and can be applied equally in developed and developing world situations. There are, however, some general differences between developed and developing world production and marketing systems that will usually mean that application in the developing world will require far greater thought and ability to get right.

Livestock generally perform a much wider range of functions in developing world farming systems than in developed world systems. Livestock are often more important to poor farmers in low input systems for providing traction, nutrient recycling (converting crop residues to manure), proving insurance (eg ability to sell animals to raise cash to pay medical expenses), asset building (being a living asset that can multiply and grow, where crops generally supply only periodic cash flow and basic food supply) and in some cases, social value. It is not uncommon that productivity and economic efficiency accounts for less than 10% of the value of livestock in such systems, in contrast to accounting for more than 90% in most developed world systems. This means that a wide range of adaptation traits, disease resistance traits and survival are much more important than productivity traits. Such traits can be difficult to measure and are more difficult to put values on than productivity traits in intensive farming systems. The emphasis here is “more difficult”. It is not impossible, but it does require more careful thought and more work to get right. Unfortunately, much more thought and effort is currently put into defining breeding objectives for livestock in developed world systems than in developing world systems. This has lead to many inappropriate breeding objectives and aggregate genotypes being defined for developing world livestock, which has been a contributing factor to the low success rate of genetic improvement programs in the developing world.

Summary

Although there are many complex issues involved in defining appropriate breeding objectives and obtaining the correct economic weights the tools exist to deal with most of these problems. Being aware of the complexity of the issues is 90% of the solution to avoiding incorrect economic weights. The general principle is that defining breeding objectives and estimating economic weights requires a thorough understanding of the economic and physical realities that shape a given production system. The economic and physical factors shaping intensive production systems are generally not that difficult to identify.

Appendix A

Useful Standard Forms of (Co) Variances and Derivatives

This is a list of some of the more useful derivations of (co)variances of simple functions that are used in the course or might be used in particular problems. Some standard derivatives also follow.

A.1 (Co) Variances

NOTATION: V = variance, W = covariance.

$V_{ax} = a^2 V_x$, where a is a constant

$V_{(x+y)} = V_x + V_y + 2W_{xy}$

$V_{(x-y)} = V_x + V_y - 2W_{xy}$

$$V_{(x_1 + x_2 + \ldots x_n)} = V_{x_1} + V_{x_2} + \ldots V_n + 2W_{x_1 x_2} + 2W_{x_{n-1} x_n}$$

$W_{ax,by} = abW_{xy}$, where a and b are constants

$$V_{\left(\frac{x}{y}\right)} = \frac{1}{\bar{y}^2} V_x + \left(\frac{\bar{x}}{\bar{y}^2}\right)^2 V_y - 2\frac{\bar{x}}{\bar{y}^3} W_{xy}$$ (1st order approximation)

$$V_{xy} = \bar{y}^2 V_x + \bar{x}^2 V_y - 2\overline{xy}\, W_{xy}$$ (1st order approximation)

$$W_{(x,xy)} = \bar{y} V_x + \bar{x} W_{xy}$$

If $T = f(x)$ and $S = f(y)$, then

$$V_T = \left(\frac{\partial T}{\partial x}\right)^2 V_x$$

and if $T = f(x_1, x_2 \ldots x_n)$ and $S = f(y_1\}, y_2 \ldots y_m)$, then

$$V_T = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial T}{\partial x_i}\right)\left(\frac{\partial T}{\partial x_j}\right) W_{x_i x_j}$$ (1st order approximation)

and $$W_{(T,S)} = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial T}{\partial x_i}\right)\left(\frac{\partial S}{\partial y_j}\right) W_{x_i y_j}$$

Note that, as always, the derivation of a variance is just a special case of derivation of a covariance. Also, both can be put in matrix notation as

$$W_{(T,S)} = \mathbf{t'Ws}$$

where **t** is a vector of length n with elements $\left(\frac{\partial T}{\partial x_i}\right)$, **s** is a vector of length m with elements $\left(\frac{\partial S}{\partial y_i}\right)$, and **W** is a n x m matrix of covariances among the x and y.

A.2 Differentiation

Recap on Basic Differentiation

A small change in a variable, x, is usually denoted as Δx or as δx. Where two variables, say x and u, are functionally related, we have

$$u = f(x)$$

where $f(x)$ denotes a function of x. A small change in x, Δx (or δx) causes a small change in u, Δu (or δu). In the limit, as Δx and hence Δu become very small, the ratio of the changes in the two variables tend to a limit

$$\frac{\Delta u}{\Delta x} \xrightarrow{\text{lim, } \Delta x \to 0} \frac{\partial u}{\partial x}$$

which is the rate of change of u with respect to x known as the differential of u with respect to x.

Some Common Differentials (All log to base e)

$u = ax^n$, $\quad \frac{\partial u}{\partial x} = anx^{n-1}$

$u = log\ x$, $\quad \frac{\partial u}{\partial x} = \frac{1}{x}$

$u = log\ v$, where $v = f(x)$ $\quad \frac{\partial u}{\partial x} = \frac{1}{v}\frac{\partial v}{\partial x}$

$u = ae^v$ where $v = f(x)$ $\quad \frac{\partial u}{\partial x} = a\frac{\partial v}{\partial x}e^v$

$$u = \frac{w}{y}, \text{ where } w = f_1(x),\ y = f_2(x) \qquad \frac{\partial u}{\partial x} = \frac{y\frac{\partial w}{\partial x} - w\frac{\partial y}{\partial x}}{y^2}$$

APPENDIX B

Example of a non-linear profit function

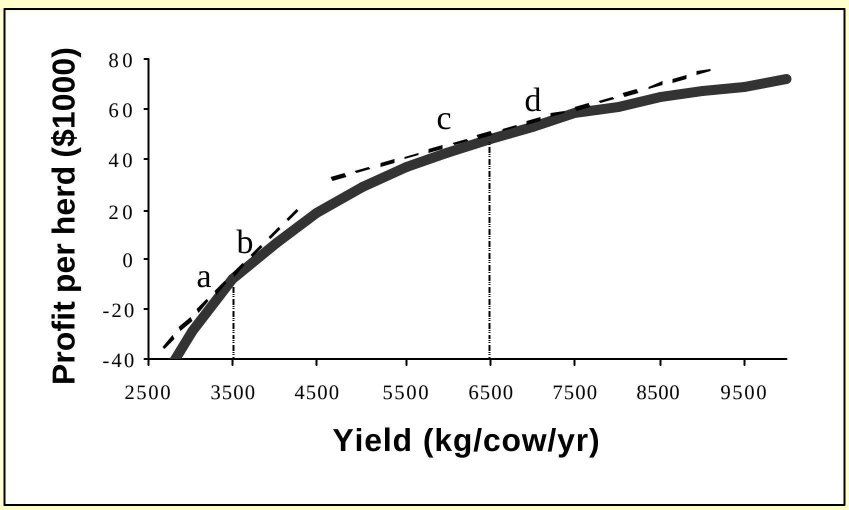
A reasonably realistic example is illustrated graphically here, with data adapted and simplified from an economic analysis of milk production in Canadian dairy cattle (Gibson, Graham and Burnside, 1992). The price of milk was 0.479 \$/kg with estimated marginal costs of production (feeding and management) of 0.093 \$/kg and an annual maintenance cost (feeding and management) of 1457 \$ per cow. There are quotas on milk production so that as milk production per cow increases, the number of cows must decrease. Imagine a herd with a quota, Q, of 300,000 kg milk and average production y kg per cow. Then the profit function from the herd can be written as

$$P = (0.479 - 0.093)Q - 1457\, n$$

where n is the number of cows, and $n = \dfrac{Q}{y}$, so that $P = 115{,}800 - \dfrac{4.371 \times 10^{8}}{y}$.

This markedly non-linear profit function is shown graphically in Figure 7.3.

Figure 7.3: Profit per herd vs. milk yield per cow with 300,000 kg quota.



The tangent to the profit function at a mean yield of 3500 kg has a much higher slope than that at 6500 kg. Since the economic weight in a linear index is given by the slope of the tangent to the profit curve, the economic weight for improving milk yield falls markedly as the mean yield increases from 3500 to 6500 kg. Quite clearly, if the range of breeding values between candidates for selection were very large (say 3000 or 4000 kg) as might happen when comparing breeds, these tangents to the profit function would do a poor job of estimating economic breeding merit. In general, linear indexes will often be poor approximations of non-linear profit functions when comparing genetic differences and should not generally be used in breed comparison work. The range of genotypes encountered in within line selection will, however, be much smaller, as illustrated below.

At any point in time the range in estimated transmitting abilities (ETA = ½ EBV) for milk yield between the top 5% and bottom 5% of bulls will be approximately 4 s_{ETA}. Assuming bulls are accurately evaluated, with r_{HI} = 1, that h^2 = 0.25, and CV = 0.18, then, approximately, s_{ETA} is given by

$$\begin{aligned} s_{ETA}^2 &= 0.25\ r_{HI}^2\ s_{g_s}^2 \\ &= 0.25\ h^2 * (CV * \bar{y})^2 \\ &= 0.25 * 0.25 * (0.18 * \bar{y})^2 \\ &= 0.002025\ \bar{y}^2, \end{aligned}$$

giving

$$s_{ETA} = 0.045\ \bar{y}.$$

(In practice s_{ETA} will be lower than this because $r_{HI} < 1$ and sires and dams of sires are intensely selected so that $s_{g_s}^2 < h^2\ s_p$).

Thus at $\bar{y}$ = 3500 kg, 95% of sire ETA will fall within the range ± 1.96 * 0.045 x 3500 = 309 kg and at $\bar{y}$ = 6500 kg, the range of sire ETA is 573 kg. These ranges are indicated by the bounds *a,b* and *c,d* on the two tangents to the profit curve in Figure 7.3.

It is now quite clear that the linear approximation given by the slope of the tangent to the profit curve gives a very good approximation to the profit function over the range of ETA encountered in practice. Indeed, Figure 7.3 gives an exaggerated example since profit is expressed at the herd level whereas it would be more realistic to express it per breeding animal (i.e. per cow) since this is the unit of genetic improvement. In this case $v = \frac{1}{n}\frac{\partial P}{\partial y}$, where *n* is the number of cows rather than $\frac{\partial P}{\partial y}$ as in Figure 7.3. In this case, since $n = \frac{g}{y}$, economic weights change at about half the rate as in the example above and the linear approximation will be an even better fit.

A (slightly) more formal argument for the use of linear indexes is as follows. In general, taking into account the non-linearity of the profit function by constructing a non-linear index, is to take into account not only the slope of the tangent to the profit curve but also the rate of change of that slope, i.e. including both $v = \frac{\partial P}{\partial y}$ and $\frac{\partial v}{\partial y} = \frac{\partial^2 P}{\partial y^2}$. In most situations $\frac{\partial^2 P}{\partial y^2}$ is a second order effect to $\frac{\partial P}{\partial y}$ so that for relatively small changes in *y*,

Δ_y, $\frac{\partial^2 P}{\partial y^2}$ may be ignored. However, it doesn't appear that any formal investigation has been made of the conditions under which ignoring terms in $\frac{\partial^2 P}{\partial y^2}$ would cause serious losses of economic progress.