Outline MT Selection

- Properties of EBVs
- Single Trait Selection Index
- Multiple Trait Selection Index

  - Predicting Response
  - Manipulating Response

  - MT Index and MTBLUP selection
  - Increased Accuracy from MT Selection
  - Effect of Incorrect Parameters
  - Other Issues

Selection Criteria → Breeding Objective

Own performance X_1

Breeding Value (s)

Performance on relatives X_2

Correlated Traits X_n

Selection Index (multiple regression)

$$EBV = Index = b_1X_1 + b_2X_2 + b_3X_3 + \ldots + b_nX_n$$

Finding the optimal index weights

Regression of A on X_i

$$\text{Regression coefficient} = \frac{\mathrm{cov}(X_i, A)}{\mathrm{var}(X_i)}$$

Examples:

X_1 = Own Performance $\qquad b = h^2$

X_1 = Dam Performance

$$b = \frac{\frac{1}{2}V_A}{V_A + V_E} = \frac{1}{2}h^2$$

Finding the optimal index weights

Regression of A on X_i

$$\text{Regression coefficient} = \frac{\operatorname{cov}(X_i, A)}{\operatorname{var}(X_i)}$$

<u>Examples:</u>

X_1 = Own Performance

$$\frac{V_A}{V_A + V_E} = h^2$$

X_1 = Dam Performance

$$b = \frac{\frac{1}{2}V_A}{V_A + V_E} = \frac{1}{2}h^2$$

Progeny Testing

$\hat{A} = b_1 P_1$

P_1 = Mean of n progeny

b_1 = Index Weight

= “heritability of progeny test

See also GEST325

Note, algebra is to illustrate not to learn

b_1 depends on the number of progeny

and on the heritability

$$b_1 = \frac{\frac{1}{2}V_A}{\frac{1}{4}V_A + (V_P - \frac{1}{4}V_A)/n} = \frac{2n}{n + 4/h^2 - 1} = \frac{2n}{n + \frac{4 - h^2}{h^2}}$$

Some basic QG: variances

X = A + E

$\mathrm{Var}(X) = \mathrm{var}(A) + \mathrm{var}(E) = V_A + V_E$

(no cov. Between A,E)

var(mean) = common variance + specific/n

X_1 = Mean of n Full Sibs

$$\mathrm{var}(X) = \frac{1}{2}V_A + V_{ce} + \frac{(\frac{1}{2}V_A + V_{te})}{n}$$

Some basic QG covariances

$X = A + E$

$\text{cov}(X_1, A) = \text{cov}(A,A) + \text{cov}(E,A) = V_A$

$\text{cov}(X_1, X_2) = a_{ij} V_A$ …….if relatives

$$\text{cov}(X_1, \bar{X}_2) = \frac{1}{n}\text{cov}(X_{1,} \sum_{i=1}^{n} X_{2i}) = \frac{1}{n}\sum_{i=1}^{n}\text{cov}(X_1, X_{2i}) = \text{cov}(X_{1,} X_{2i})$$

Finding the optimal index weights

Selection Index (multiple regression)

EBV = Index = $b_1X_1 + b_2X_2 + b_3X_3 + \ldots\ldots + b_nX_n$

Regression of A on X; X'=[X1, X2,.... Xn]

$$\frac{\mathrm{cov}(X, A)}{\mathrm{var}(X)} = \mathrm{var}(X)^{-1}\,\mathrm{cov}(X, A) = P^{-1}G$$

$$P = \mathrm{var}(X)$$

$$G = cov(X, A)$$

Selection index with more information sources
(multiple regression)

X = vector with phenotypes (“P_i values” = sel. criteria)

A = True breeding Value

var(X) = P = matrix = $$\begin{bmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1, X_2) \\ \mathrm{cov}(X_2, X_1) & \mathrm{var}(X_2) \end{bmatrix}$$

cov(X,A) = G = vector = $$\begin{bmatrix} \mathrm{cov}(X_1, A) \\ \mathrm{cov}(X_2, A) \end{bmatrix}$$

Opt. weights: b = $P^{-1}G$ = “cov/var_X” (single trait Sel Index eqn: Pb = G)

Example: $X_{1=}$ weight for own phenotype

X_2= mean of n full sibs

	n=3		n=10	
h^2	b_1	b_2	b_1	b_2
0.10	0.09	0.12	0.08	0.32
0.30	0.26	0.26	0.22	0.49
0.50	0.43	0.29	0.38	0.48
0.70	0.62	0.24	0.57	0.36

- Own performance more important with high heritability
  ……..and smaller amount of family info
- Otherwise, family information more important

Accuracy of selection index

r_{IA} = correlation between Index and A

$$= \frac{cov(I,A)}{\sigma_I \, \sigma_A} = \frac{\sigma^2_I}{\sigma_I \, \sigma_A} = \underline{\sigma_I} / \sigma_A = \sqrt{(b'Pb/V_A)}$$

because cov(I,A) = cov(bX,A) = b'cov(X,A)

= b'G = b'Pb = b'var(X)b

= var(bX)

= var(I)

Note: I = EBV = sel. criterion

A = BV = Objective

→ r_{IA} = sqrt(var(EBV) / V_A)= accuracy = correlation

Summary of this lecture

- Selection Index Theory can be used to work out weights and accuracy for a given set of information about an particular EBV
- Quantitative Genetic Theory and matrices (P, G) are used to work out such index weights (b) and accuracies
- In Genetic Evaluation we use BLUP where this all occurs 'automatically'
- Selection Index Theory still useful to predict what happens
  - Accuracy for a given amount of information
  - Importance of own vs family information for given situations