Matrix Algebra

Matrix Algebra: Key points

- Dimensions: Rows x Columns

cows

observation

$$\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$$

Matrix Algebra: Key points

- Multiplication

Nr. of goods

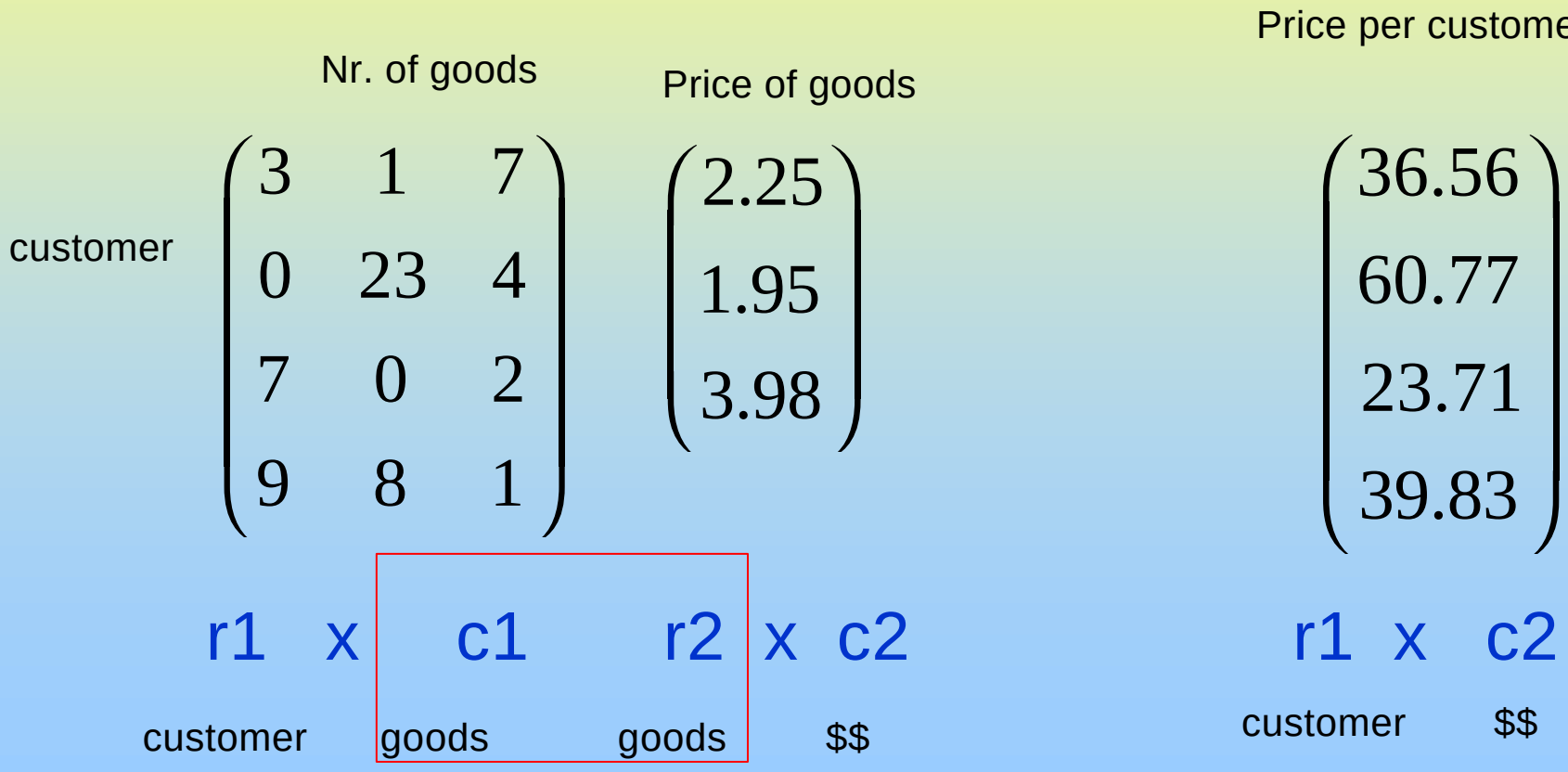
customer $\begin{pmatrix} 3 & 1 & 7 \\ 0 & 23 & 4 \\ 7 & 0 & 2 \\ 9 & 8 & 1 \end{pmatrix}$

Price of goods $\begin{pmatrix} 2.25 \\ 1.95 \\ 3.98 \end{pmatrix}$

Price per customer $\begin{pmatrix} 36.56 \\ 60.77 \\ 23.71 \\ 39.83 \end{pmatrix}$

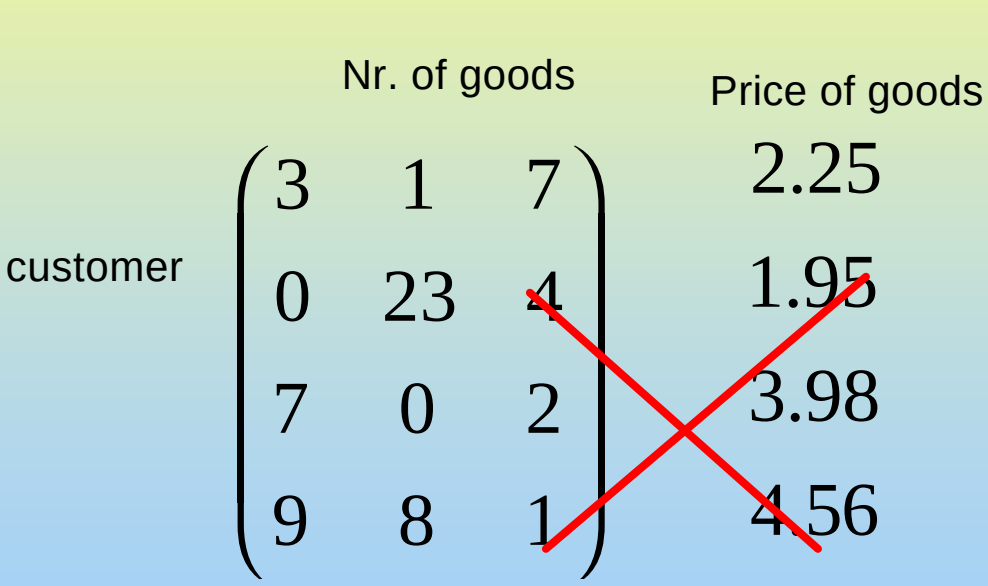
Matrix Algebra: Key points

- Multiplication: is it possible?



Matrix Algebra: Key points

- Multiplication: is it possible?



Price per customer

r1 x c2

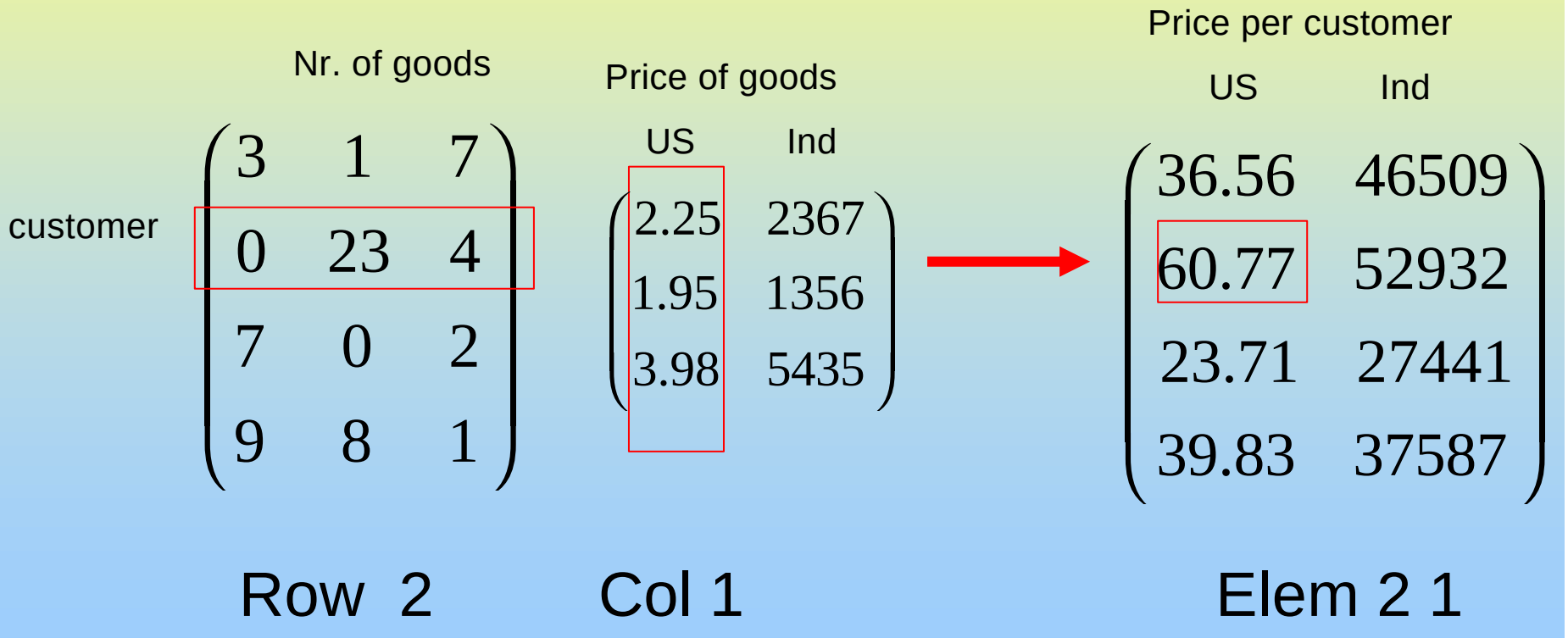
customer $$

Matrix Algebra: Key points

- Multiplication: row x column

Matrix Algebra: Key points

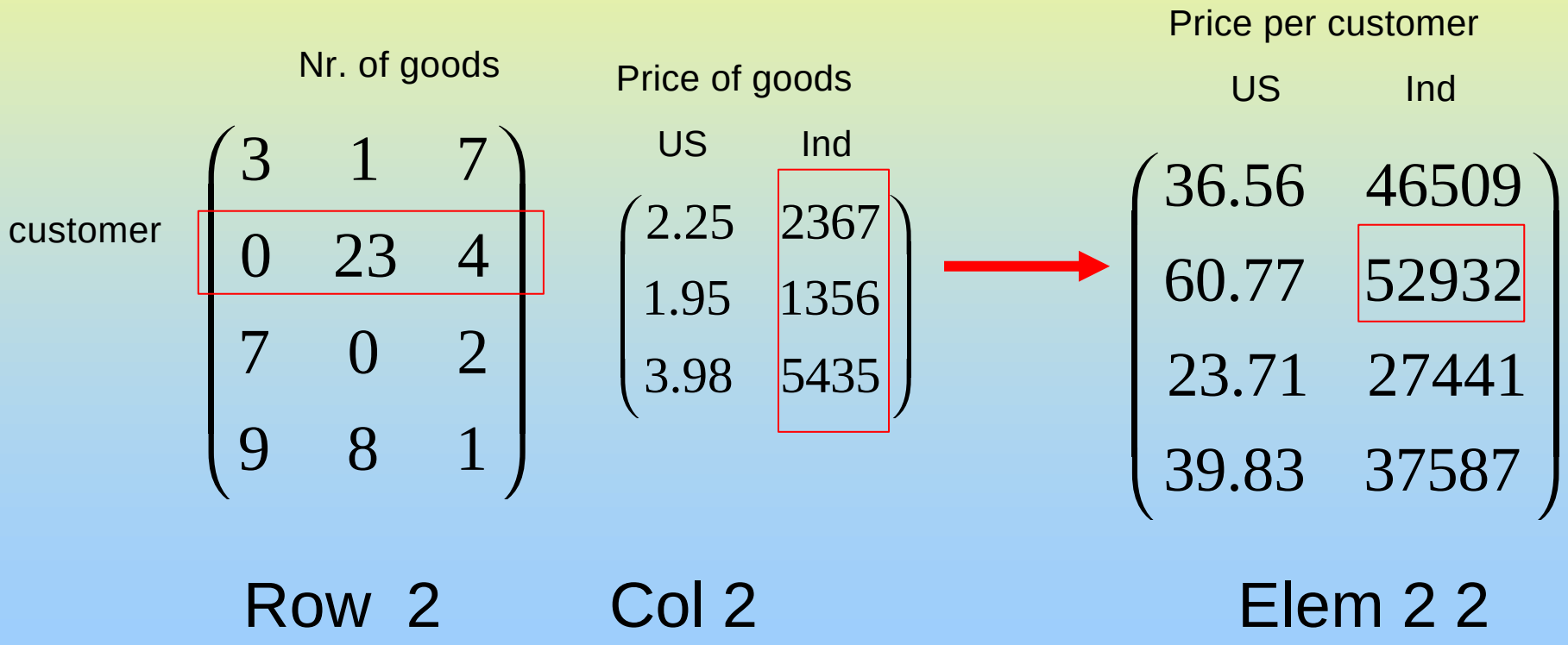
- Multiplication: row x column



Row 2 Col 1 Elem 2 1

Matrix Algebra: Key points

- Multiplication: row x column



Row 2 Col 2 Elem 2 2

Matrix Algebra: Key points

Incidence matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & & \\ & & \\ & & \end{pmatrix}$$

transpose

Matrix Algebra: Key points

Incidence matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 & \\ & & \\ & & \end{pmatrix}$$

transpose

Matrix Algebra: Key points

Incidence matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

transpose

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 3 \end{pmatrix}$$

Matrix Algebra: Key points

Incidence matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 3 \\ & 2 & \\ & & \end{pmatrix}$$

transpose

Matrix Algebra: Key points

Incidence matrix

X'

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

transpose

X

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

=

Numbers per class

X'X

$$\begin{pmatrix} 5 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix}$$

Matrix Algebra: Key points

Incidence matrix

X'

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

transpose

data

y

$$\begin{pmatrix} 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{pmatrix}$$

=

totals

X'y

$$\begin{pmatrix} 50 \\ 17 \\ 33 \end{pmatrix}$$

Matrix Algebra: Inverse?

Numbers per class

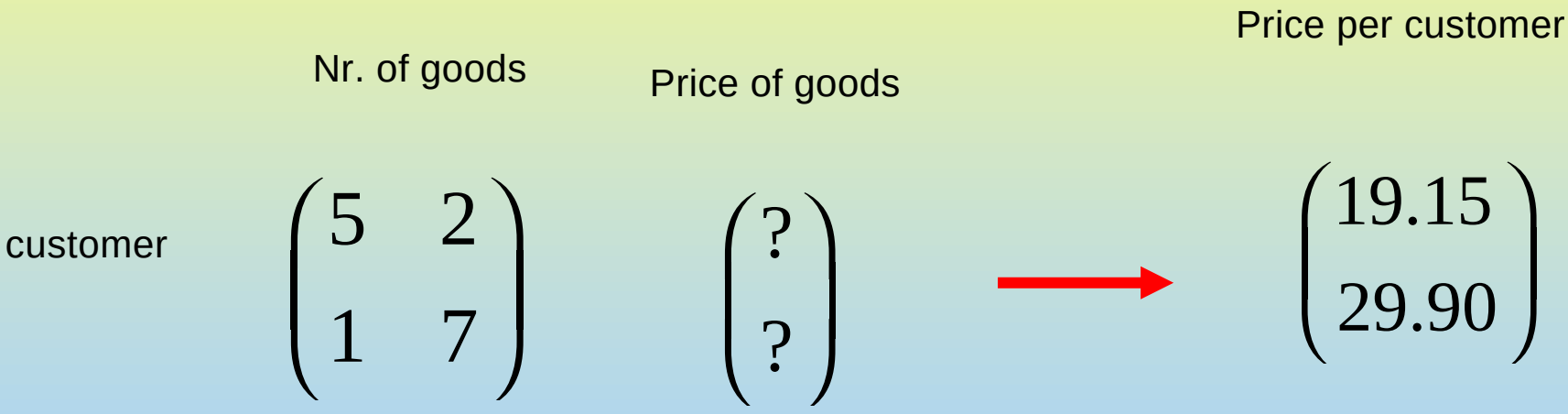
X'X

totals

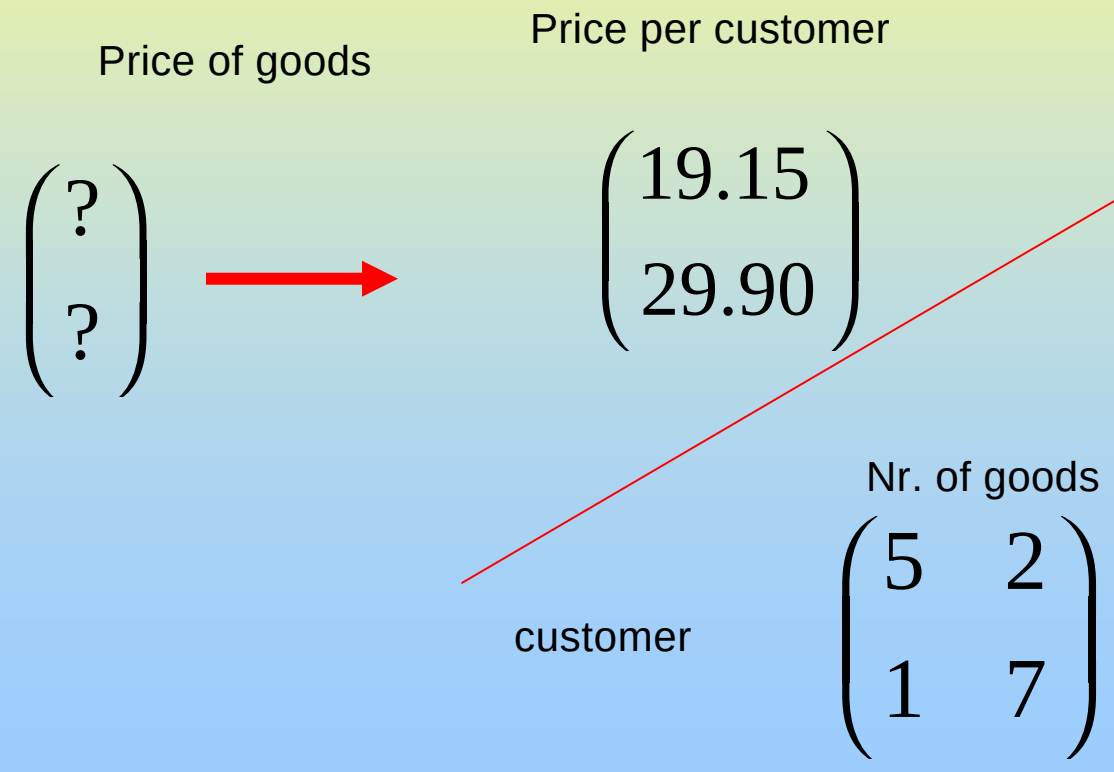
X'y

$$\begin{pmatrix} 5 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 50 \\ 17 \\ 33 \end{pmatrix}$$

Matrices representing equations



Matrix Algebra: Key points



Matrix Algebra: Key points

Price of goods

$$\begin{pmatrix} ? \\ ? \end{pmatrix} \longrightarrow \begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}^{-1} \quad \text{Price per customer} \quad \begin{pmatrix} 19.15 \\ 29.90 \end{pmatrix}$$

- For inverse to exist, matrix must be square

Inverse ~ Substitution

Price of goods

Price per customer

$$\begin{pmatrix} ? \\ ? \end{pmatrix} \rightarrow \begin{pmatrix} .2121 & -.0606 \\ -.0303 & .1515 \end{pmatrix} \begin{pmatrix} 19.15 \\ 29.90 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}^{-1}$$

$=(5-(2*1)/7)^{-1}$

$=(7-(2*1)/5)^{-1}$

Not all matrices have an inverse

Effects per class

Numbers per class

totals

$X'X^{-1}$ $X'y$

$$\begin{pmatrix} mean \\ a1 \\ a2 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 50 \\ 17 \\ 33 \end{pmatrix}$$

Matrix Singularity

X

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

X'X

$$\begin{pmatrix} 5 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix}$$

r1 = r2 + r3

Det(X'XX)=0

Inverse does not exist

No unique solution

More unknowns than independent equations

Covariance matrices

- Covariance matrix

$$\begin{matrix} 34 & 3 & 14 \\ 3 & 56 & 57 \\ 14 & 57 & 78 \end{matrix}$$

Correlation matrix

$$\begin{matrix} 1 & .0688 & .2719 \\ .0688 & 1 & .8624 \\ .2719 & .8624 & 1 \end{matrix}$$

- Symmetric
- Consistent → Eigenvalues > 0

Covariance Matrix Consistency

Covariance matrices

$$\begin{pmatrix} 100 & 3 & 14 \\ 3 & 100 & 85 \\ 14 & 85 & 100 \end{pmatrix} \qquad \begin{pmatrix} 100 & 78 & 14 \\ 78 & 100 & 85 \\ 14 & 85 & 100 \end{pmatrix}$$

Eigenvalues

$$\begin{pmatrix} 99 & 14 & 187 \end{pmatrix} \qquad \begin{pmatrix} -9 & 86 & 223 \end{pmatrix}$$