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DYNAMIC OPTIMIZATION FOR EVALUATING EXTERNALITIES IN AGROFORESTRY SYSTEMS:

An example from Australia

1. INTRODUCTION

Integrating trees in cropping and grazing systems – agroforestry – can provide many benefits in the Australian agricultural context. These include the production of timber and non-timber products such as oils and flowers, fodder, windbreak protection, shade and shelter, wildlife habitat, flood mitigation, soil-erosion control, improved water quality, and reduced dryland-salinity emergence. Prinsley (1992), Cleugh, Prinsley, Bird, Brooks, Carberry, and Crawford et al. (2002) and other authors in the same volumes present some examples for Australia. Many of the benefits from trees are off-farm environmental services, which are public goods or externalities and which landholders may not take into account. The social benefits from trees may therefore exceed the private benefits, and given such a divergence, landholders may conserve and plant too few trees from a social perspective. Without perfect information, landholders may even underestimate their private benefits from trees and further exacerbate this divergence. Concern for this issue is not new in Australia (see Tisdell, 1985).

Market failure due to externalities and imperfect information provides a rationale for government intervention to encourage landholders to invest in vegetation management and reforestation. Regulatory, extension, and market-based approaches are all being used to this end by governments in Australia.

State Governments have established regulatory controls on land clearing of private native vegetation (Walpole, 1999; Stoneham, Chaudhri, Ha, & Strappazzon, 2003), such as the Native Vegetation Conservation Act in New South Wales (NSW) and The Planning and Environment Act in Victoria. State and Federal Governments have also implemented extension programs to provide funds and/or assistance to landholders and community groups to manage native vegetation on private land. At the State level, these programs include Land for Wildlife and Trust for Nature in

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Victoria, and the Voluntary Conservation Agreement Program in Queensland. At the Federal level, there is Landcare, One Billion Trees, Save the Bush, Bushcare, National Heritage Trust, and the National Action Plan for Salinity and Water Quality. Stoneham et al. (2003) provides an overview of some of these programs.

State and Federal governments are also trialing market-based approaches to managing environmental externalities. For example, the NSW Government is trialing an Environmental Services Scheme where landholders can apply for funding to change their land-use practices to improve any of six environmental services (carbon-sequestration, biodiversity, salinity, soil, water, and acid-sulfate benefits). In Victoria, the Government is trialing a BushTender initiative through which landholders ‘bid’ to conserve areas of private native vegetation for biodiversity enhancement (Stoneham et al., 2003). The Commonwealth, State, and Territory Governments are also jointly funding a National Market-Based Instruments Pilots Program to trial the use of trading instruments and offset schemes to change the behavior of landholders towards important natural-resource management issues, such as biodiversity, dryland and irrigation salinity, and water quality.

An evaluation of the economic efficiency of these three approaches is beyond the scope of this chapter. We emphasize here that, whichever approach is used, it is important to understand the environmental benefits that can be provided by trees. Lack of information about the environmental benefits from trees is one of the obstacles to restoring landscape vegetation to address land-degradation issues in Australia. When making tree/crop decisions, landholders tend to focus on the relative input and output prices of crops and trees, as well as risk considerations. Moreover, the long time lag between investment and returns in the forestry enterprise generally makes this activity unattractive compared with alternative land uses when not all the benefits are considered.

In this chapter, we demonstrate a general technique for evaluating externalities provided by trees for crops through improvements in land productivity from mitigation of land degradation. This extends the analysis presented by Cacho (2001) for an agroforestry system in a catchment affected by dryland salinity. In the analysis presented here we also include payments for carbon sequestration by the forestry enterprise. A general model of an agroforestry system comprising an annual crop and a tree crop is initially developed for a catchment represented by a homogenous plot of land under the control of a single-decision maker or managed as common property. An application is then presented, using a dynamic optimization approach to estimate optimal land allocation between forestry and agriculture.

2. MODEL AND METHODOLOGY

The optimal management of an agroforestry system in the presence of land degradation is addressed by the decision question: what is the optimal area of trees to plant in an agroforestry system and how long should the trees be kept before harvest? In essence, the question boils down to estimating the direct (timber) and indirect (prevention of land degradation, carbon sequestration) net benefits provided by trees and comparing them to the net benefits provided by crops. The decision is made in a dynamic setting, where decisions taken today have a bearing on the range of decisions available in the future.

The problem of the socially-optimal forest rotation in the presence of non-timber values in forests was first studied by Hartman (1976). He derived the decision rule (with an infinite planning horizon) for the optimal rotation when the standing forest provides positive externalities. He showed that the optimal harvest age is that at which the growth rate of the forest equals the value of the discounted stream of non-timber benefits relative to timber benefits up to the time of harvest. Hartman's work has been extended by authors such as Bowes and Krutilla (1985, 1989), Englin and Klan (1990) and Swallow, Parks, and Wear (1990).

Bowes and Krutilla (1985) focus on the multiple-use management of public forestlands, where land managers must consider not only the value of timber harvested, but also non-market benefits such as recreation, water flow, and wildlife. They introduce multiple forest stands of different ages and the manager's decision is whether to harvest some or all of them at a given point in time. Bowes and Krutilla (1989) present a detailed review of previous models as well as an applied forest-management model where the aesthetic, water flow, and amenity values are expressed as functions of the age of the forest stand. They also show how to apply linear programming to solve multiple-use forest management problems.

In the case of agroforestry, timber production and non-timber benefits may occur in different areas of land, where the area planted to crops benefits from the area planted to trees. This results in an additional decision variable, not considered in the papers discussed above; the proportion of the area available that should be planted to trees, with the remaining area planted to crops.

2.1. General model

Consider a homogeneous plot of land that can be planted to any combination of trees and crops, and where trees provide land conservation services. The benefit obtained from a hectare of land over a single forest rotation of length T is:

$$NPV(k,T) = (1-k)\sum_{t=1}^{T} a_t(s_t,k)\cdot(1+r)^{-t} + k\sum_{t=1}^{T} f_t(s_t,k)\cdot(1+r)^{-t} - k\cdot c_E \,; \quad \text{with } 0 \le k \le 1 \qquad (1)$$

where s_t is the state of the land in year t, k is the proportion of the plot planted to forest and c_E is the cost of forest establishment. The state variable, s_t, represents a quality indicator, such as soil depth, soil-carbon content and soil fertility; or it may be defined to represent a negative quality, such as soil salinity or sodicity. Note that this equation considers only establishment costs per hectare, so the cost of forest establishment increases linearly with the proportion of land under trees. In other words, no fixed costs that may cause economies of scale are considered.

The net present value (*NPV*) of the flow of benefits obtained over a single forest rotation (equation 1) consists of the accumulation of direct monetary benefits, $a(\cdot)$, provided by an annual agricultural crop, and the benefits provided by a forestry operation, $f(\cdot)$. The discount rate is represented by r.

The dynamics of the state variable are given by:

$$s_{t+1} = s_t + \Delta s_t\left(s_t, k\right) \tag{2}$$

with s_0 given.

The expected signs of the key derivatives are:

$$\frac{\partial a_t}{\partial s_t}, \frac{\partial f_t}{\partial s_t} \begin{cases} < 0 \text{ if } s_t \text{ measures land degradation} \\ > 0 \text{ if } s_t \text{ measures land quality} \end{cases}$$

$$\frac{\partial a_t}{\partial k} \begin{cases} < 0 \text{ if trees compete with crops for resources} \\ = 0 \text{ if the only effect of trees on crops is through } \Delta s_t \text{ (equation 2)} \\ > 0 \text{ if trees benefit crops directly (i.e., other than through } \Delta s_t\text{)} \end{cases}$$

$$\frac{\partial f_t}{\partial k} \begin{cases} < 0 \text{ unlikely} \\ = 0 \text{ if } k \text{ does not affect forest growth (other than through } \Delta s_t\text{)} \\ > 0 \text{ if there are economies of scale in forest establishment and growth} \end{cases}$$

Since, s_t is the state variable, which represents some measure of land quality, if s_t refers to a 'negative quality' such as salinity then an increase in s_t will lead to a decrease in returns from agriculture and forestry. Conversely, if the measure refers to a 'positive quality' such as soil fertility, then an increase in s_t will lead to an increase in returns from agriculture and forestry.

The reasoning behind the signs of the derivatives with respect to k is as follows: trees may compete with crops for light, water, and nutrients, so a larger area of trees may result in lower crop yields per unit area ($\partial a/\partial k < 0$); but trees may also benefit crops directly, by providing services such as shelter, nitrogen fixation, and pest control ($\partial a/\partial k > 0$). If both effects are present then the ultimate direction of change depends on the balance between positive and negative interactions. Important indirect services are also provided by trees, such as prevention of soil erosion and salinity, in these cases it is possible to have $\partial a/\partial k = 0$. This means that the effects of trees on crops occur indirectly, through changes in land quality (equation 2) and the resulting changes in crop yields, as represented in equation 4 below.

The relative area of trees in an agroforestry system may also affect the net benefits obtained from the forestry enterprise. Although changes in tree density can affect the growth rate of individual trees, this does not apply in our case because we assume constant planting density; so the case where $\partial f/\partial k < 0$ is unlikely to occur in the context of the problem studied here[1]. A larger area of trees may be associated with larger returns from forestry ($\partial f/\partial k > 0$) if there are economies of scale. In the case study presented later we assume that both derivatives with respect to k are zero, so the area planted to trees affects land quality only indirectly (through equations 2, 4 and 6).

The net monetary benefits obtained from agriculture in any given year t are:

$$a_t = p^a y_t^a - c^a \qquad (3)$$

where p^a is the price of the crop and c^a is the cost of crop production per hectare. y_t^a is crop yield obtained in year t and is affected by the productivity of the land as follows:

$$y_t^a = \bar{y}^a\, Q_t^a(s_t) \qquad (4)$$

where $\bar{y}^a$ is expected yield under 'normal' land productivity and Q_t^a is the land-productivity function for agriculture.

The annual net monetary benefits of the forestry operation are given by:

$$f_t = p^f y_t^f - c_t^f \qquad (5)$$

where the price of forest outputs (p^f) is assumed to be constant irrespective of the age of the forest, while costs (c_t^f) and yields (y_t^f) do depend on forest age. Yields are also affected by land productivity:

$$y_t^f = \bar{y}_t^f \, Q_t^f(s_t) \tag{6}$$

where $\bar{y}_t^f$ is expected yield in year *t* and Q_t^f is the land-productivity function for forestry. Because the model is solved in discrete time, c_t^f can be conveniently represented as a vector of known annual values rather than as an explicit function of time. The annual yields of the forestry operation, y_t^f , can also be represented as a vector (Cacho, 2001). In the numerical model developed below, a forest-growth function is used to estimate both annual forest outputs and final harvest. The model allows carbon-sequestration services (the annual forest output) to produce revenues. The forest-growth function is based on Cacho, Hean, and Wise (2003).

In the numerical analysis presented later, the economic model (1) is first solved in simulation mode, where the NPV is estimated for any arbitrary combination of input values, this is useful to gain a general understanding of how expected profits are affected by the decision variables and the initial state of the land. The model can also be solved in optimizing mode, by finding the values of *k* and *T* that maximize the value of (1) subject to the constraints imposed by equations (2) to (6).

Cacho (2001) showed the derivation of the first-order conditions for maximization of equation 1 with respect to *k*, while keeping *T* constant at the recommended tree-harvest age. Some of the relationships in the model are nonlinear and have complex derivatives. This means that it is more convenient to undertake direct numerical maximization of the objective function, using any of a number of constrained optimization techniques, than to use the first order conditions which require derivatives to be estimated.

The *NPV* function (1) represents the first forestry cycle only, so it does not take account of the opportunity cost of keeping trees on the ground rather than harvesting them. Cacho et al. (2003) used an infinite-rotation model to deal with this problem in their carbon-sequestration model. This common approach to including the opportunity cost of not harvesting is not applicable here, because land quality changes over time. This means that subsequent forestry cycles are not identical to the first cycle and optimal cycle length and optimal forest area may change between cycles. So, with forestry cycles denoted by *n*, the optimization problem is:

$$\max_{k_n, T_n} V_0 = \sum_{n=1}^{\infty} NPV(s_n, k_n, T_n) \cdot (1+r)^{-(T_0 + \ldots + T_{n-1})} \tag{7}$$

subject to equations (1) to (6) and with $T_0 = 0$. This is a very complex problem with an infinite number of decision variables. Fortunately, thanks to Bellman's principle of optimality, it can be simplified to a two-decision variable problem to be solved recursively. So we convert the multiple-rotation problem into a dynamic programming (DP) problem, where the stages are forestry cycles of length T_n. The DP recursive equation is:

$$V_n(s_n) = \max_{(k_n, T_n)} \left(NPV(s_n, k_n, T_n) + V_{n+1}(s_{n+1}) \cdot (1+r)^{-T_n} \right). \tag{8}$$

subject to equations (1) to (6). The problem is solved for an infinite planning horizon, by backward induction (Kennedy, 1986), until convergence in V_n is achieved. This involves combining the DP algorithm with the numerical model described below.

2.2. Numerical model

Dryland salinity is a major land degradation problem in Australia. It has been caused by replacing perennial native vegetation with farming and grazing systems that allow a larger proportion of rain to recharge groundwater systems, and is evidenced by high and rising saline water tables in low-lying, discharge areas of catchments. Greiner and Cacho (2001) provide an overview of the problem.

An issue that has received some attention recently, although it has not been debated in the formal literature, is the possibility of payments for carbon-sequestration services to contribute to salinity mitigation (see Hean, Cacho, & Menz, 2003). Although the main focus in the global-warming debate is on emissions (sources), sinks, such as carbon-sequestration in trees, have a role to play. Trees remove carbon dioxide from the atmosphere during photosynthesis and store the carbon in wood, leaves and roots; while the oxygen is released back into the atmosphere. At the international level, the Kyoto Protocol has provided much of the impetus for the policy debate, but other policies exist at different government levels. In Australia, both Victoria and NSW have enacted legislation to allow for the separate ownership of land, trees, and carbon-sequestration rights to facilitate carbon-credit exchanges, and they have also implemented schemes to investigate the potential for carbon-credit markets.

The numerical model below is based on dryland-salinity emergence in Australia in the presence of carbon-sequestration payments. Control of the land-degradation problem is driven by forest growth, defined as:

$$\Delta b_t = \left(\alpha_G \cdot b_t^{\beta_G} - \gamma_G \cdot b_t \right) \cdot Q_t^f \tag{9}$$

where b_t is aboveground forest biomass, Δb_t is the annual biomass increment, and Q_t^f is the land-productivity function for forestry as previously defined. α_G, β_G and γ_G are parameters determined by tree species and site characteristics, in this study we use parameter values for *Eucalyptus nitens* in southern Australia (Table 1). Biomass is measured in metric tons per hectare (t/ha).

The forestry operation is assumed to have two types of outputs: annual outputs (carbon-sequestration services) that depend on forest biomass accumulation, Δb_t, and a final harvest (timber) that depends on stemwood volume, v_T, measured in cubic meters per hectare (m^3/ha). Assuming 50 percent of forest biomass is carbon (Brown, 1997; Hamburg, 2000), annual forest output is proportional to carbon-sequestration rate. Hence, equations (5) and (6) are together replaced by:

$$f_t = p_c \cdot 0.5 \cdot \Delta b_t - c_t^f \text{ ; for } 0 < t < T \qquad (10)$$

for annual forest outputs, and by:

$$f_T = p_c \cdot 0.5 \cdot \Delta b_t + p_v \cdot v_T - p_c \cdot 0.5 \cdot b_T \qquad (11)$$

for final harvest. p_c and p_v are the prices of carbon (Australian$/ton of Carbon [$/t C]) and timber (Australian$/cubic meter [$/m^3]) respectively. In the second term in equation 11, which represents the value of the timber harvest, p_v is assumed to be constant, although more realistically it would depend on the stemwood diameter of the trees at harvest (see Cacho et al., 2003).

The first term in both equations (10) and (11) represents the annual payments from carbon sequestered in the interval (0, …, T), while the last term in equation 11 represents the assumption that carbon credits received during forest growth have to be fully redeemed upon harvest. Full debit at harvest means that the total amount of carbon credits received during the life of the forest must be paid back to the investor by the landholder at harvest. As pointed out by Cacho et al. (2003), this implicitly assumes that the contract ends as the sequestered carbon is no longer under the control of the landholder. In other words, the contract between an investor (e.g., a power company) and a landholder to capture and maintain a given amount of carbon out of the atmosphere expires when the forest is harvested, and the landholder cannot guarantee that the terms of the contract will continue to be fulfilled. Once the contract expires, the investor would have to find an alternative sequestration project, or pay a carbon tax. This scheme is equivalent to the rental carbon market proposed by Marland, Fruit, and Sedjo (2001).

Biomass and timber volume are estimated by numerical integration of the model as follows:

$$b_{t+1} = b_t + \Delta b_t \qquad (12)$$

$$v_{t+1} = v_t + \Delta v_t \qquad (13)$$

with b_0 and v_0 given. Increments in timber volume are estimated by:

$$\Delta v_t = \frac{0.7 \cdot \Delta b_t \cdot \left(\frac{b_t}{\theta_G} \right)^{0.2}}{d} \tag{14}$$

where d is wood density (t/m^3) and θ_G is the maximum biomass achievable by the forest (t/ha):

$$\theta_G = \left(\frac{\alpha_G}{\gamma_G} \right)^{1/(1-\beta_G)} \tag{15}$$

Equation 14 assumes that an increasing proportion of new growth is stemwood. When trees are young, they generally have more branches and foliage relative to stem than old trees. By maturity, approximately 70 percent of biomass is stemwood.

Where dryland-salinity emergence is a problem, trees can be strategically placed in recharge areas to reverse trends in rising water tables. In this example, land quality (s_t) in equation 2 is represented by the depth of the water table (w_t) measured in meters (m) below the soil surface. Hence, the land-productivity function is defined as:

$$Q_t^j = 1 - \alpha_j \cdot \exp\left(-\beta_j \cdot w_t\right) \text{ for } j=a,f \tag{16}$$

where the parameters α_j and β_j depend on land and plant characteristics. This function implicitly accounts for the relationship between the water table and salinity, and the effect of salinity on crop yields and tree growth (see Cacho, 2001 for an interpretation of this function).

Annual changes in the water-table depth are given by:

$$\Delta w_t = -\frac{(1-k)R_t^a + k\,R_t^f}{\gamma_R} \tag{17}$$

where R_t^j represents the amount of recharge associated with activity j ($j=a,f$) in year t and γ_R converts total recharge (in mm/m^2) to water-table depth changes (in m) over the watershed. The value of γ_R depends on characteristics of the aquifer and the nature of water movements below the soil surface, and can be adjusted to represent areas with different levels of propensity to dryland-salinity emergence.

Recharge rates depend on the amount of rainfall received, the amount of runoff to streams and the amount of water taken up by plants. Young trees do not eliminate deep water, but as they grow larger their roots reach deeper into the water table and eliminate large volumes of water through evapotranspiration, so R_{tf} becomes negative as trees grow. The recharge rates associated with agriculture and forestry are given by:

$$R_t^a = \alpha_{RA} - \beta_{RA} \cdot \frac{y_t^a}{\overline{y}^a} \tag{18}$$

$$R_t^f = \alpha_{RF} - \beta_{RF} \cdot b_t \tag{19}$$

where α_{RA} is the recharge (millimeters/ year [mm/yr]) that would occur under fallow land and α_{RF} is the recharge under newly planted tree seedlings. β_{RA} and β_{RF} are parameters, and the remaining variables are as previously defined.

The model just described can be simulated for any given tree area and forestry cycle-length by solving equations (9) to (19) while numerically integrating the water table as follows:

$$w_{t+1} = w_t + \Delta w_t(w_t, k) \tag{20}$$

Equation 20 replaces equation 2 in this example, since land quality (s_t) is represented by the depth of the water table.

The model was simulated for the base-case parameter values shown in Table 1. These parameters represent good-quality land in Australia. The tree-growth parameters are for *Eucalyptus nitens* (Cacho et al., 2003), and the salinity parameters are largely based on Cacho (2001), but the expected crop yield, $\overline{y}^a$, is higher to represent good-quality land (i.e., 4.5 t/ha). Currency ($) is measured in Australian dollars, and biomass and carbon (C) weights are measured in metric tons per hectare (t/ha).

Four scenarios (Table 2) were simulated, including the base case, to explore the effects of carbon payments and discount rates on optimal solutions. There is a lot of uncertainty about the price of carbon that may emerge from a competitive carbon market. The carbon price used here has been used in other studies (see Cacho et al., 2003) and is conservative.

2.3. Implementation of the DP model

The application of dynamic programming (DP) models to forestry is not new. Kennedy (1986) devotes a chapter to the derivation of optimal forestry-rotation rules in a DP context. He describes both deterministic and stochastic models, where optimal thinning and harvesting rules are derived based on the age of the forest. In the simplest version (no thinning), the decision variable is binary: keep the trees, or harvest and replant. When thinning is introduced the decision variable becomes continuous, representing the proportion of the forest harvested at a given age, so total harvest is represented by a value of one and no thinning has a value of zero. Although the decision variable is continuous within the interval (0-1), it is generally expressed as a set of discrete values in DP models. Because of problems with dimensionality, these models are usually solved at time steps of 5 or 10 years. Kennedy (1986) presents a summary of DP applications to forestry management published between 1966 and 1982.

Solution of a DP model consists of solving a recursive equation that maximizes the reward obtained from managing the system. The reward in this case is the net present value of the agroforestry system, and the decision variables are the area of trees to plant (k) and the rotation length (T). The recursive equation is:

$$V_n(s_n) = \max_{(k_n,T_n)}\left(NPV(w_n,k_n,T_n) + V_{n+1}(w_{n+1})\cdot(1+r)^{-T_n}\right) \qquad (21)$$

The DP model for the salinity problem was implemented by solving this equation recursively, and backwards in time, starting from time period T+1. The model is solved for a discrete set of values of k, T and w and an optimal decision rule is derived. The simulation model represented by equations (1), (3), (4), and (9) to (20) was solved for values of k ranging from 0 to 1 at increments of 0.01, T from 1 to 50 years at increments of one year and w from 1.8m (shallow) to 4.0m (deep) at increments of 0.1m. So there are 22 states (w), resulting in 484 (or 22×22) possible state transitions, $w_t \rightarrow w_{t+1}$, to be controlled by 101 possible values of k and 50 possible values of T.

The recursive equation 21 can be solved by interacting directly with the simulation model. This involves running a simulation for each possible combination of k and T and finding the (k,T) values that result in each of the 484 possible state transitions. However, the direct-simulation approach is not efficient, because the same set of simulations is run at every stage n of the DP algorithm as the recursive equation is solved backwards in time.

Table 1. Parameter values used in the numerical model. (Currency in Australian$).

Parameter	*Value*	*Units*	*Description*	*Equation*
Biophysical				
α_G	4.189	1/yr	forest growth parameter	9, 15
β_G	0.681	*	forest growth parameter	9, 15
γ_G	0.595	1/yr	forest growth parameter	9, 15
θ_G	453.976	t/ha	maximum forest biomass	14, 15
d	0.7	t/m^3	wood density	14
α_a	3.684	*	land-quality parameter, crop	16
β_a	2.608	*	land-quality parameter, crop	16
α_f	1.5	*	land-quality parameter, tree	16
β_f	2.608	*	land-quality parameter, tree	16
α_{RA}	80	mm/yr	crop recharge intercept	18
β_{RA}	40	mm/yr	crop recharge slope	18
α_{RF}	55	mm/yr	tree recharge intercept	19
β_{RF}	0.7	mm/yr	tree recharge slope	19
γ_R	160	mm/m	recharge conversion factor	17
$\bar{y}^a$	4.5	t/ha	maximum crop yield	4, 18
Economic				
r	6, 12	%	discount rate	1, 8
p^a	180	$/t	price of crop output	3
p_v	30	$/m^3	price of timber	11
p_c	20	$/t	price of carbon	10, 11
c^a	140	$/ha	variable cost of crop	3
c_E	2000	$/ha	forest establishment cost	1
c^f_t	50	$/ha	forest maintenance cost	5, 10

**coefficient has dimension 1.*

Table 2. Scenarios simulated. (Currency in Australian$).

Scenario	*Carbon price ($/t)*	*Discount rate (%)*
1 (base)	0	6
2	20	6
3	0	12
4	20	12

We follow a more efficient approach, consisting of creating and saving all the relevant matrices once, by running 101×22×22 simulations, each representing 50 years of forest growth. The saved matrices can be reloaded at any time to solve the DP algorithm. Using this approach it took about 1/6th of the time to solve each DP problem as compared with direct simulation.

The DP approach has the advantage that it yields not only an optimal solution for a particular scenario, as other dynamic optimization methods do, but it also produces an optimal decision rule based on the state of the system at any time. The optimal decision rule can be repeatedly applied to derive optimal (state and decision) paths through time, without having to solve a new optimization problem for each initial state.

3. RESULTS AND DISCUSSION

This section presents the results of both simulation and optimization. A typical simulation run is implemented by solving equation 1 for a set of arbitrary values of the decision variables k and T and for a given initial value of the state variable (w_0). The simulation run produces a state trajectory (w_t) for the given assumptions. Useful insights into the workings of the agroforestry system are obtained by running simulations. However, it is virtually impossible to find the optimal decision trajectory by trial and error with the simulation model. So the DP algorithm described in the previous section is also solved, to obtain a set of sequential values of k_n and T_n that maximize the present value of the agroforestry system for the given initial conditions. The optimization model was solved for an infinite planning horizon, but only 100 years of results are presented, sufficient time to determine whether the system is sustainable.

3.1. Simulation

Figure 1 shows simulation results for a single forestry cycle for selected combinations of initial water-table depth (w_0) and forest area (k). With no trees (k=0), the water table rises over time to reach the soil surface (w_t decreases to zero) by year 15 (Figure 1A). This trend in w_t results in decreasing crop yields, particularly after year 10 (Figure 1B). By year 15 crop yields drop to zero as the land becomes irreversibly lost to conventional agriculture.

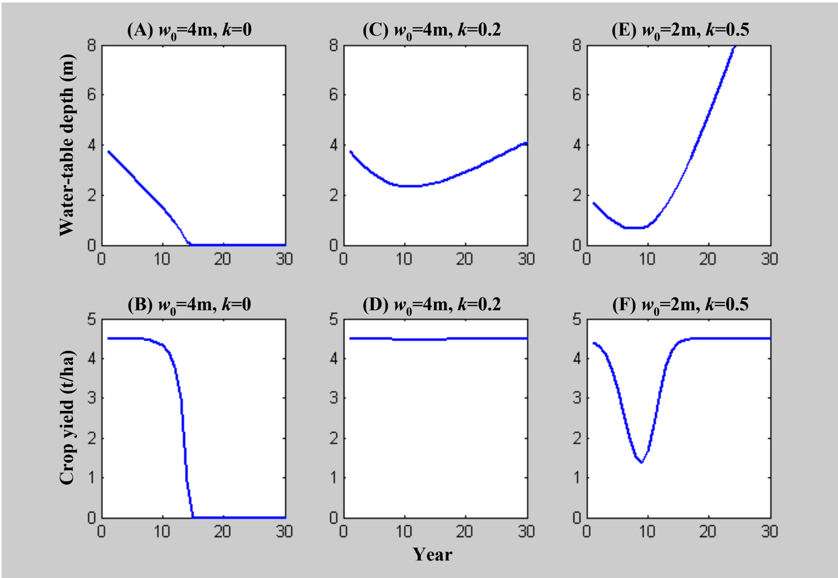

Figure 1. Simulation results: trajectory of the water-table depth (A, C and E) at three different combinations of initial conditions (w_0) and tree area (k), and the corresponding trajectories of crop yields (B, D and F).

Planting 20 percent of the area to trees (k = 0.2) causes the water table to stabilize at around 4m depth by year 30 (Figure 1C); this results in stable crop yields through time (Figure 1D). When the initial water table is close to the soil surface (2m), a tree area of 0.2 is not enough to achieve sustainable crop yields; so it is necessary to set k=0.5 to control salinity emergence (Figure 1E). In this case, crop yields decrease initially (Figure 1F), as the water table approaches the surface (as w_t approaches zero), but they recover after year 9 to regain their original value (4.5 t/ha) by year 15.

The benefits provided by trees take time to become evident (Figure 2). The maximum *NPV* at base parameter values is obtained with k = 0.15 in year 50 (Figure 2A). If one were constrained to a planning horizon of five years or less, crop monoculture would be more profitable than agroforestry. This is clear from the negative slope of *NPV* with respect to k (in Figure 2A) during the early years. Carbon payments help somewhat in making forestry more profitable and make it more attractive to keep trees longer (Figure 2B), but the maximum *NPV* occurs at the same k as with no carbon payments.

As the water table approaches the soil surface (as w_t approaches zero), an interesting twist is introduced (Figures 2C and 2D). With w_0 = 2m, the value of k at which maximum *NPV* occurs is now 0.5 (compared with 0.15 in the base case). Furthermore, at values of k less than 0.4, *NPV* decreases if the planning horizon is extended beyond five years as crop yields are reduced by salinity (Figure 4C). This effect is similar but more pronounced with carbon payments (Figure 4D). The abrupt increase in *NPV* as *k* increases between 0.4 and 0.6, in Figures 4C and 4D, is where the positive externality provided by trees for crops is more pronounced. In this region, marginal increases in tree area reverse the salinization process to a level that maintains crop yields.

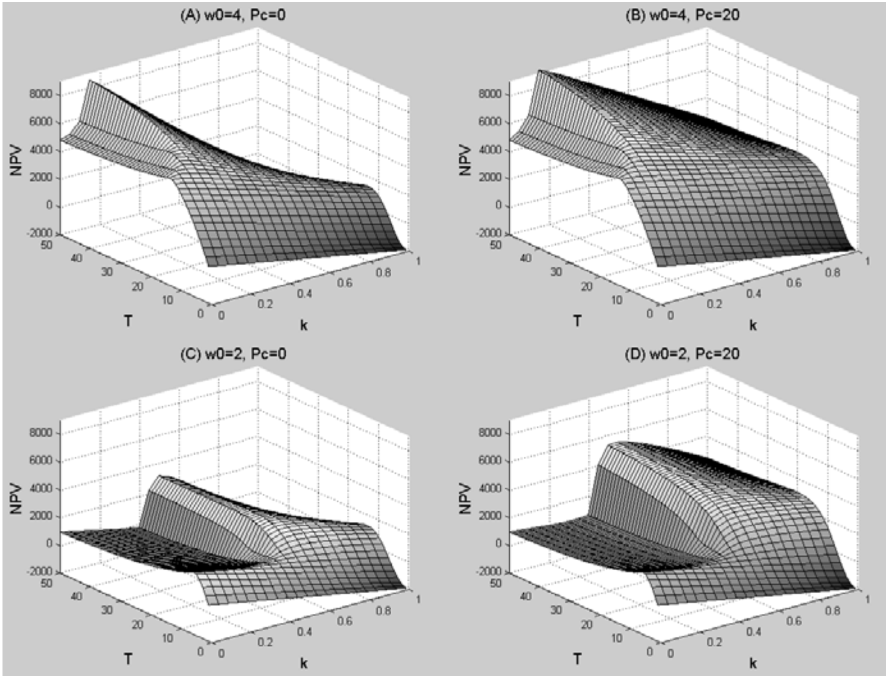

Figure 2. Simulation results: net present value (equation 1) of the agroforestry operation with respect to tree area (k) and cycle length (T). Two initial water-table depths (w_0 = 4m and w_0 = 2m) and two carbon prices (p_c=\$0/t and p_c=\$20/t) are shown.

These simulation results are interesting and help us understand the economics of the salinization problem; but they are based on a single forestry cycle and, as discussed before, do not take into account the opportunity cost of keeping trees on the ground. The multiple-cycle problem is addressed by solving the DP problem (equation 21) for an infinite planning horizon in the following section.

3.2. Optimization

The DP model was solved for the base parameters in Table 1 and the four scenarios in Table 2. As explained earlier, these matrices were saved and later used to solve the DP model and perform post-optimality analysis.

The DP solutions for the four scenarios considered are presented in Figure 3. The optimal decision rule (k^*,T^*) is shown for discount rates of 6 percent (Figures 3A and 3B) and 12 percent (Figures 3C and 3D).

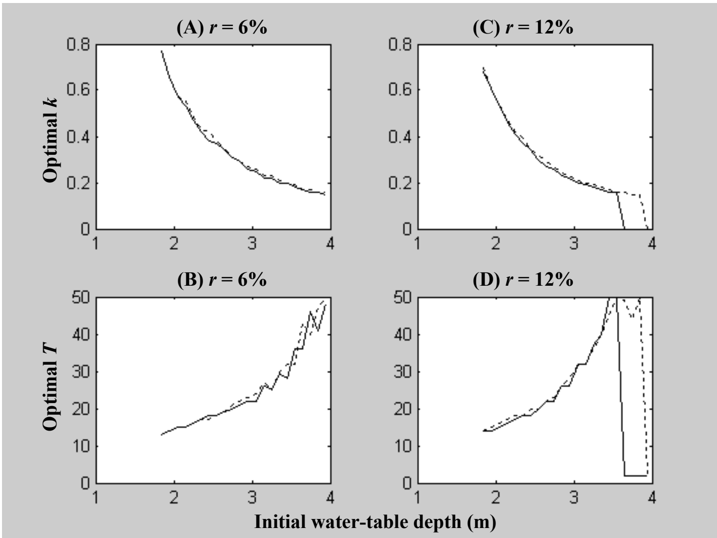

Figure 3. Optimization results obtained by solving the DP model for four scenarios. The solid lines represent cases with no carbon payments, dotted lines represent cases with carbon payments of $20/t.

At low values of w_t (near the soil surface), it is optimal to plant most of the land to forestry (Figure 3A) with a relative short cycle (Figure 3B); at w_t = 2m, for example, the optimal control (k^*,T^*) = (0.61, 14). But this changes as the water-table depth increases below the soil surface; so at w_t = 3.5m, (k^*,T^*) = (0.2, 32).

A similar pattern for the optimal control (k^*,T^*) is observed at the high discount rate (Figures 3C and 3D), but with slightly lower areas planted to forestry (k^* = 0.57 and 0.16 for w_t = 2m and 3.5m respectively). Interestingly, increasing the discount rate does not affect the optimal cycle length at low values of w_t (T^* remains at 14 years), but causes it to increase at high values of w_t (T^* = 50 years when w_t = 3.5m in Figure 3D, compared to 32 years in Figure 3B). An important effect of the high discount rate is that forestry drops out of the optimal solution at water-table depths further than 3.5m below the soil surface, i.e., at $w_t > 3.5$m, $(k^*,T^*) = (0, 0)$ in Figures 3C and 3D.

The optimal state transitions, given by the changes in the water-table depth ($w_t \rightarrow w_{t+1}$) resulting from applying the optimal decision rule, are shown in Figure 4.

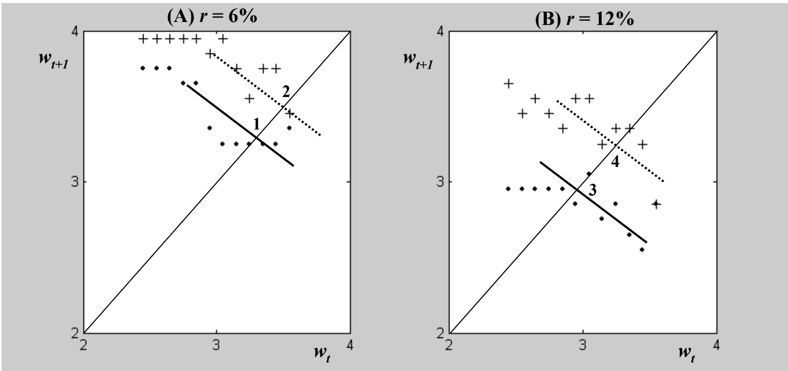

Figure 4. Optimal state transitions for the water table depth at two different discount rates and with (dotted line) or without (solid line) carbon payments. The 45° line starting from the origin represents the steady state.

The optimal state transitions (Figures 4A and 4B) do not result in smooth lines with respect to w_t, because of the discrete nature of the solution procedure and the consequent introduction of rounding and truncation errors; so optimal state transitions are presented as points for the relevant set of w_t values in the state vector. Both sets of points show a distinctive downward trend over the relevant interval of w_t values. These trends are indicated by negatively-sloped segments, one for each scenario. The 45-degree lines starting at the origin represent the steady state, where $\Delta w_t = 0$m. The intersection between the optimal state transition and the steady-state line marks the optimal 'target' water-table depth in the long run. These target w_t values are: 3.2m, 3.5m, 2.9m, and 3.2m, for scenarios 1, 2, 3, and 4, respectively (Figures 4A and 4B). Hence the optimal equilibrium water-table depth is positively related to carbon price and negatively related to the discount rate.

The optimal value function (V in Figure 5) is the present value of the current water-table depth when the system is managed according to the optimal decision rule in perpetuity. The value of V increases at a decreasing rate as w_0 increases in depth below the soil surface. The pattern is the same for all four scenarios, but the position of the curve changes depending on the discount rate and price of carbon.

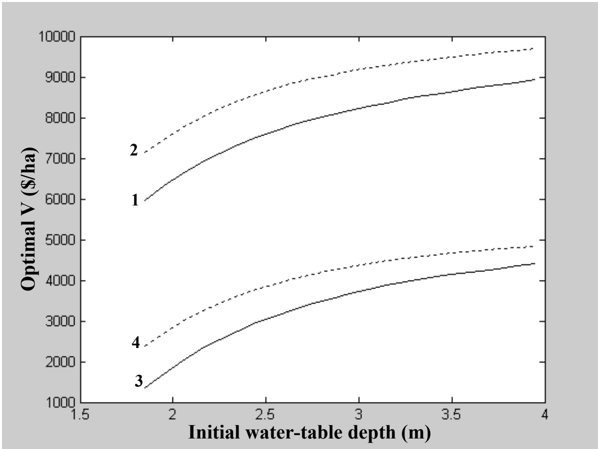

Figure 5. Optimal value function for the four scenarios.

The shadow price of the water table is a useful measure, because it indicates the dollar value of an improvement in land quality (deeper water table). This value is in present-value terms, so it takes account of changes in the productive capacity of the land in perpetuity. The shadow price is equivalent to the costate variable (λ) used in the discrete version of the maximum principle in optimal control theory (see Kennedy, 1988). The shadow price is defined as $\lambda_0 = dV/dw_0$. This derivative was estimated numerically from the DP results shown in Figure 5. As would be expected, the value of λ_0 is high at low values of w_0 (poor land quality), implying that the marginal benefit of an improvement in land quality is high (over $3,000/ha at w_0 = 2.0m). The value of λ_0 decreases as land quality improves, to reach about $500/ha at w_0 = 3.5m.

The shadow prices in Figure 6 are not affected by the discount rate within the range tested, but they decrease when carbon payments are introduced. This means that the marginal value of land improvement is lower when carbon payments are available. The reason for this is not immediately obvious, but will become clear later when carbon stocks are discussed.

Further insight into optimal results can be gained by analyzing the optimal paths for selected initial conditions. The optimal paths discussed below were derived by applying the optimal decision rules (Figure 3) and simulating the system as explained in the methodology section.

3.3. Optimal Paths

Figure 7 shows the water-table paths that result from applying the optimal decision rule for each scenario over a period of 100 years. Two different initial states (w_0 = 2m and w_0 = 4m) are presented. The target w_t values, derived in Figures 4A and 4B above, correspond to the ends of forestry cycles. Each cycle starts with a decrease in w_t (as the water table approaches the surface), followed by an increase (as the water table becomes deeper) as trees grow and absorb more water. Each forestry cycle ends at a peak w_t (at values of between 2.6m and 4m depending on the discount rate and carbon price). These peaks represent the target w_t values discussed above.

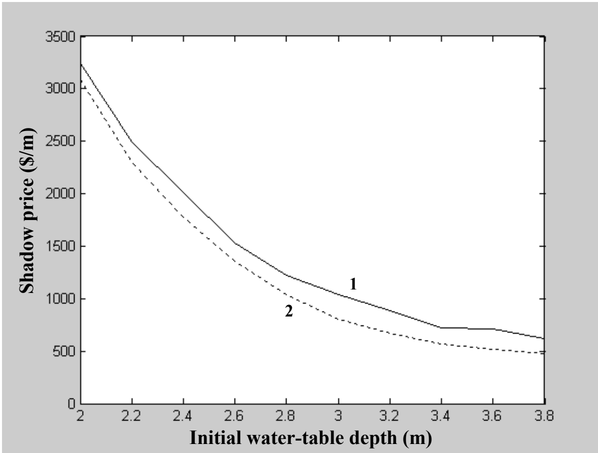

Figure 6. Shadow price of the water table for scenarios 1 and 2.

With w_0 = 2m, the presence of carbon payments makes the second and subsequent cycles longer (dotted lines in Figures 7A and 7C); but the first cycle is not affected by carbon price. This is because the priority in the first cycle is to fix the shallow water-table problem before it is too late, so a large area of forest is planted regardless of whether carbon payments are received.

With w_0 = 4m, the situation is different (Figures 7B and 7D). With a deeper water table (4m), there is no pressure to fix the problem immediately. So it is optimal to run down the system for the first 20 years. After year 20, preventive action starts having an effect on land quality and w_t starts increasing in depth for the rest of the first forestry cycle. Carbon payments have an interesting effect at the low discount rate (Figure 7B). In this case carbon payments do provide an incentive for land conservation, and the optimal value of w_t at the end of the first cycle (around year 50) is 3.5m below the soil surface with carbon payments (scenario 2) as compared to 2.5m deep without carbon payments (scenario 1). This result suggests that, in situations where salinity emergence is not imminent, carbon payments do make a difference in encouraging early preventive action. This occurs only at the low discount rate.

With w_0 = 4m and the higher discount rate (Figure 7D), the story changes. Carbon payments (scenario 4) do not have much of an effect on w_t during the first 40 years as compared with the case where no carbon payments are made (scenario

3). The higher discount rate (12 percent) gives more weight to the value of the early crops and discounts future productivity losses more heavily, so it is optimal to run down the system for a little longer before taking preventive action.

A striking result, which becomes apparent when comparing Figures 7A and 7C against 7B and 7D, is the strong path dependency on initial conditions that the problem exhibits during the first 100 years. The initial water-table depth dramatically influences the optimal state path. Eventually though, the optimal paths obtained for different initial states will tend to converge towards the target w_t explained in connection with Figures 4A and 4B.

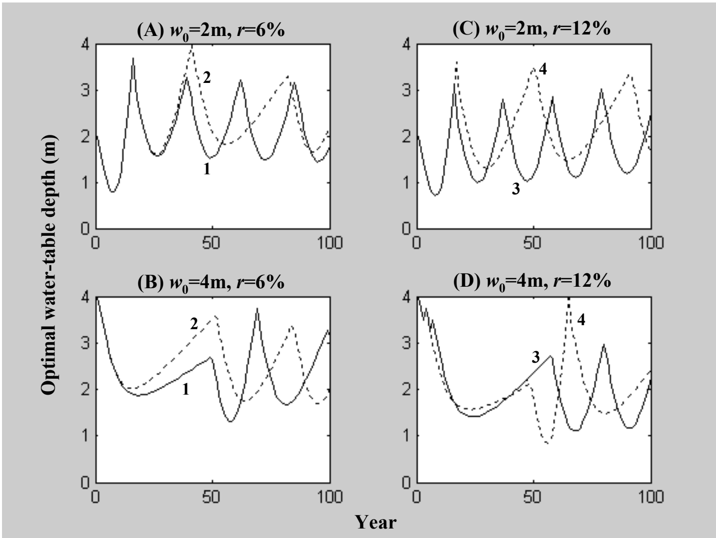

Figure 7. Optimal state paths at two different discount rates and two carbon prices. Numbers next to the curves represent the four scenarios considered.

The optimal control paths that cause the state paths depicted in Figure 7 are presented in Table 3. With w_0 = 2m, the general trend consists of a large area of forestry (mean 0.55) and a short rotation (mean 15.3 years) established in the first cycle; followed by smaller forest areas (between 0.22 and 0.26 on average) with longer rotations (between 25 and 31 years on average) in subsequent cycles. With w_0 = 4m (Table 3), the trend is different; a small area of trees is planted in the first cycle (mean 0.08), and larger areas of forest with longer rotations are established in later cycles (mean k^* = 0.24 and mean T^* = 26.5 years in cycle 6).

Something that is masked when comparing only the means is the interaction between discount rates and initial water-table depth. At w_0 = 4m, the optimal control (k^*, T^*) for the first cycle is around (0.15, 50) for scenarios 1 and 2, compared to (0, 2) for scenarios 3 and 4.

The cumulative *NPV* increases at a decreasing rate with subsequent cycles (Table 3, bottom), because of discounting. By cycle 6 the present value of future cycles is negligible. These results suggest the amount by which land prices should decrease as land degradation sets in. Comparing the results obtained with w_0 = 2m ($6,173) against those obtained with w_0 = 4m ($8,940) suggests that, under perfect competition in a deterministic world, the better quality land should be worth about $2,667 more per hectare.

Table 3. Optimal control paths and the resulting cumulative profits over six forestry cycles for four scenarios at two initial water-table depths.

Cycle	w_0=2m					w_0=4m				
	Optimal tree area (k^*)									
	Scenario					Scenario				
	1	2	3	4	Mean	1	2	3	4	Mean
1	0.57	0.57	0.53	0.53	0.55	0.15	0.16	0.00	0.00	0.08
2	0.25	0.26	0.30	0.21	0.26	0.34	0.19	0.00	0.15	0.17
3	0.25	0.17	0.30	0.18	0.23	0.20	0.23	0.16	0.53	0.28
4	0.25	0.23	0.30	0.19	0.24	0.22	0.21	0.27	0.17	0.22
5	0.26	0.21	0.26	0.19	0.23	0.25	0.19	0.26	0.20	0.23
6	0.25	0.19	0.27	0.18	0.22	0.26	0.21	0.27	0.20	0.24
	Optimal cycle length (T^*, years)									
	Scenario					Scenario				
	1	2	3	4	Mean	1	2	3	4	Mean
1	15	15	15	16	15.3	48	50	2	2	25.5
2	22	24	20	32	24.5	19	32	2	44	24.3
3	22	40	20	40	30.5	29	25	50	16	30.0
4	22	25	20	36	25.8	26	30	22	45	30.8
5	22	30	22	36	27.5	22	32	22	32	27.0
6	22	32	22	40	29.0	22	30	22	32	26.5
	Cumulative NPV (AUS$/ha)									
	Scenario					Scenario				
	1	2	3	4		1	2	3	4	
1	2,748	3,548	1,133	2,032		8,470	9,190	1,132	1,132	
2	5,230	6,398	1,624	2,728		8,769	9,626	2,035	4,914	
3	5,917	7,293	1,676	2,748		8,910	9,686	4,661	4,926	
4	6,107	7,364	1,681	2,748		8,934	9,701	4,667	4,930	
5	6,158	7,382	1,682	2,748		8,939	9,704	4,668	4,930	
6	6,173	7,385	1,682	2,748		8,940	9,704	4,668	4,930	

3.4. Carbon stocks

One remaining question in regard to the optimal paths is what happens to carbon stocks. The optimal water-table paths discussed above are associated with optimal biomass paths as trees grow during a forestry cycle. These biomass paths can be expressed in terms of carbon (Figure 8) by multiplying biomass by 0.5.

The optimal carbon path (Figure 8) reinforces the findings in the previous section, that with w_0 = 2m, land restoration practices (a large area of trees) are adopted early (Figures 8A and 8C); whereas with w_0 = 4m, the system is exploited during the first cycle to produce a larger area of crops, with only a small area of trees being planted (Figures 8B and 8D). The early exploitation of the system leads to the need to restore land quality by the second cycle, requiring a larger area of trees to be planted in year 50 (solid line in Figure 8B).

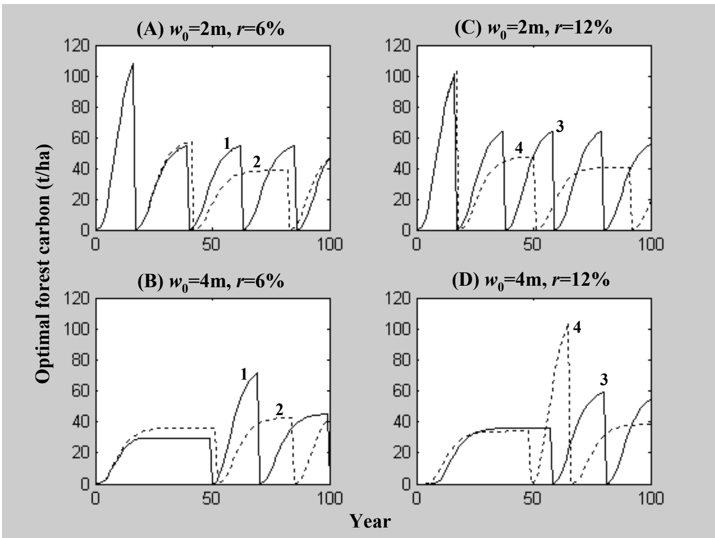

Figure 8. Optimal carbon stock paths.
Numbers next to the curves represent the four scenarios considered.

The average carbon stocks (or time-averaged carbon) can be used to assess whether carbon payments influence the amount of carbon that is sequestered by the agroforestry system. The average amount of carbon fixed in the forest over the first 100 years is presented in Table 4 for each scenario. Carbon payments and discount rates have only a small effect on the average amount of carbon stocks. Optimal carbon stocks range between 30.3 and 32.2 tC/ha, on average, for the four scenarios. In contrast, initial water-table depth has more of an effect on carbon stocks, with averages of 33.7 and 28.6 tC/ha for the cases with w_0 = 2m and w_0 = 4m respectively. This is a 15 percent drop in optimal carbon stocks as the initial water-table depth increases from 2 to 4 meters below the soil surface. This explains the result obtained in Figure 6, where the shadow prices of w_0 were lower in the presence of carbon payments than in their absence. Higher values of w_0 result in lower optimal carbon stocks (Table 4), because there is less need for land restoration. Hence, in the presence of carbon payments, the benefit of lower salinity is slightly reduced by a decrease in carbon payments received, because of reductions in the biomass carbon stock.

Table 4. Optimal forestry carbon stocks (tC/ha) for eight scenarios. Only aboveground biomass of trees is considered.

	r=6%		*r=12%*		
w_0	p_c=0	p_c =20	p_c =0	p_c =20	*Mean*
2	33.1	32.9	36.1	32.5	33.7
4	28.3	27.8	28.3	29.9	28.6
Mean	30.7	30.3	32.2	31.2	

The foregoing analysis is based on land of excellent quality by Australian standards. Land of lower quality can be represented by reducing expected crop yields (parameter $\overline{y}^a$) and tree-growth parameters (α_G, β_G, γ_G). This was not done here, as it would have added little to the analysis, given that the purpose of this chapter is to demonstrate the application of the techniques available to analyze externalities in agroforestry systems, and not to provide policy prescriptions.

Finally, it must be pointed out that, although the model developed in this chapter has been applied to the problem of dryland salinity in Australia, its applicability is much broader. The analytical techniques illustrated here can be applied to virtually any land degradation problem that involves trees and crops. The key to adapting the model to other problems resides in estimating two equations: i) the state-transition as a function of the control variables *k* and *T*; and ii) the land productivity function $Q(s_t)$ which describes the effect of land quality on crop and tree growth. In our example, these functions are represented by equations 2 and 16. Equation 16 implicitly represents the effect of salinity on yields; the shape of this function probably holds for many land-degradation problems, but different functional forms may be appropriate in some cases (Swallow et al., 1990; Swallow, Talukdar, & Wear, 1997).

4. CONCLUDING COMMENTS

In this chapter we have developed a model of an agroforestry system where trees and crops interact. The model was used within a dynamic programming algorithm to determine the optimal area of trees to plant and the optimal length of forest cycles in an Australian example. A numerical model of dryland-salinity emergence was solved for a homogeneous plot of land under the management of a single decision-maker. Therefore the only externalities considered were those that trees provide for crops, through mitigation of land degradation. The negative externalities that upstream landholders may impose on downstream landholders were not considered.

An important finding of our numerical analysis was the optimal decision rule, whereby forest area and cycle length depend on initial land quality. When initial land quality is poor, it is optimal to plant a large area of forest and follow a short cycle. As land quality improves, it becomes optimal to plant a smaller area of trees and keep the trees for a longer period. The main driver of this decision is salinity emergence. In general, payments for carbon-sequestration services had little influence on the optimal solution. Although they increased landholder profits, they did not tend to increase the optimal amount of carbon stored in the forest. It must be pointed out, however, that only aboveground biomass carbon was considered in the payment scheme, soil carbon was not included.

The model developed here was applied to a salinity problem in Australia, but the general approach is widely applicable to other land-degradation problems and locations. The two main limitations of our model are that it is deterministic and that it assumes a homogeneous plot of land under single management. The former limitation is important, because of the variability of rainfall, which is the main driver of water-table recharge that leads to salinity emergence. Also, given the possible irreversibility of severe salinity, it is important to account for the probability that high-recharge events will occur. The latter limitation becomes relevant only if we wish to extend the analysis to account for interactions among landholders, with heterogeneous land, who may participate in market-based solutions to land-degradation problems.

Our dynamic-programming approach has advantages for making the model stochastic. In this regard, the state-transition matrix can be replaced by a set of transition-probability matrices associated with sets of decisions. The DP algorithm can then be used to estimate expected profits. So the main problem is to obtain sufficient data to estimate probabilities that certain events (such as rainfall, yields, and prices) will occur.

Relaxing the assumption that the land is homogeneous would require a more complex model; one that includes different types of land as well as the interaction between upstream and downstream plots. This may cause a dimensionality problem that is difficult to handle with DP, particularly if the model is also stochastic.

5. NOTES

[1] A reviewer pointed out that low-density planting may lead to lower growth and tree quality and hence to lower tree values, particularly if trees are intimately mixed. These effects are not considered in our model.

6. REFERENCES

Bowes, M. D., & Krutilla, J. V. (1985). Multiple use management of public forestlands. In A. V. Kneese & J. L. Sweeney (Eds.), *Handbook of natural resources and energy economics*, Vol. 2 (pp. 531-569). Amsterdam: Elsevier Science Publishers.

Bowes, M. D., & Krutilla, J. V. (1989). *Multiple management: The economics of public forestlands*. Washington, D.C.: Resources for the Future.

Brown, S. (1997). *Estimating biomass and biomass change in tropical forests: A primer*. Rome: Food and Agriculture Organization of the United Nations.

Cacho, O. J. (2001). An analysis of externalities in agroforestry systems in the presence of land degradation. *Ecological Economics, 39*, 131-143.

Cacho, O. J., Hean, R. L., & Wise, R. (2003). Carbon-accounting methods and reforestation incentives. *Australian Journal of Agricultural and Resource Economics, 47*, 153-179.

Cleugh, H. A., Prinsley, R., Bird, P. R., Brooks, S. J., Carberry, P. S., & Crawford, M. C. (2002). The Australian National Windbreaks Program: overview and summary of results. *Australian Journal of Experimental Agriculture, 42*(6), 649-664.

Englin, J. E., & Klan, M. S. (1990). Optimal taxation, timber and externalities. *Journal of Environmental Economics and Management, 18*, 263-275.

Greiner, R., Cacho, O. J. (2001). On the efficient use of a catchment's land and water resources: dryland salinization in Australia. *Ecological Economics, 38*, 441-458.

Hamburg, S. P. (2000). Simple rules for measuring changes in ecosystem carbon in forestry-offset projects. *Mitigation and Adaptation Strategies for Global Change, 5*, 25-37.

Hartman, R. (1976). The harvesting decision when a standing forest has value. *Economic Inquiry, 14*, 52-58.

Hean, R. L., Cacho, O. J., & Menz, K. (2003). Farm forestry, carbon-sequestration credits and discount rates. In T. W. Graham, D. J. Pannell, & B. White (Eds.), *Dryland Salinity: Economic Issues at Farm, Catchment and Policy Levels* (pp. 133-144). Cooperative Research Centre for Plant-Based Management of Dryland Salinity, University of Western Australia, Perth.

Kennedy, J. O. S. (1986). *Dynamic programming: applications to agriculture and natural resources*. London: Elsevier Applied Science.

Kennedy, J. O. S. (1988). Principles of dynamic optimization in resource management. *Agricultural Economics, 2*, 57-72.

Marland, G., Fruit, K., & Sedjo, R. (2001). Accounting for sequestered carbon: the question of permanence. *Environmental Science and Policy, 4*, 259-268.

Prinsley, R. T. (1992) The role of trees in sustainable agriculture – an overview. *Agroforestry Systems, 20*, 87-115.

Stoneham, G., Chaudhri, V., Ha, A., & Strappazzon, L. (2003). Auctions for conservation contracts: an empirical examination of Victoria's BushTender trial. *Australian Journal of Agricultural and Resource Economics, 47*(4), 477-500.

Swallow, S. K., Parks, P. J., & Wear, D. N. (1990). Policy-relevant nonconvexities in the production of multiple forest benefits. *Journal of Environmental Economics and Management, 19*, 264-280.

Swallow, S. K., Talukdar, P., & Wear, D. N. (1997). Spatial and temporal specialization in forest ecosystem management under sole ownership. *American Journal of Agricultural Economics, 79*, 311-326.

Tisdell, C. A. (1985). Conserving and planting trees on farms: lessons from Australian cases. *Review of Marketing and Agricultural Economics, 53*, 185-194.

Walpole, S. C. (1999). Assessment of the economic and ecological impacts of remnant vegetation on pasture productivity. *Pacific Conservation Biology, 5*, 28-35.